Guidelines to select boundary conditions for mode shapes in flexible multibody systems

GAMM 2010, March, 24th 2010

Dr.-Ing. Andreas Heckmann, DLR
Institute of Robotics and Mechatronics

Deutsches Zentrum für Luft- und Raumfahrt e.V.
in der Helmholtz-Gemeinschaft

Contents

Pre-processing of flexible bodies in MBD

- Modelling of the structure
  - analytically, PDE, FEM
- Choice of Boundary Conditions
- Solution of the eigenvalue problem
- Selection of an reduced set of modes
- optional: Generation of additional modes
  - e.g. additional particular solutions of the PDE, e.g. static modes, frequency response modes, thermal response modes
- Modal transformation
- Consideration of the non-linear reference motion
  - floating frame of reference formulation

see e.g. Wallrapp94, Argawal/Shabana95, Dietz99

Boundary Conditions I: Beam

- PDE of the Euler-Bernoulli-beam with separation approach

$$\ddot{y}(x,t) + \frac{EI}{\varrho A}\, y''''(x,t) = 0 \qquad\qquad y(x,t) = w(x)\cdot q(t)$$

- spatial differential equation

$$w'''' + \lambda^4 w = 0 \qquad w(x) = e^{\nu x}$$

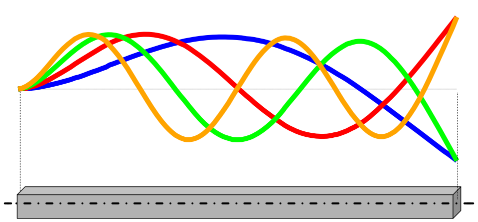
$$w(x) = c_1 \cosh(\nu x) + c_2 \sinh(\nu x) + c_3 \cos(\nu x) + c_4 \sin(\nu x)$$

- 4 boundary conditions

kinematic (essential) BC

$$w(x_b = 0) = 0$$

$$w'(x_b = 0) = 0$$



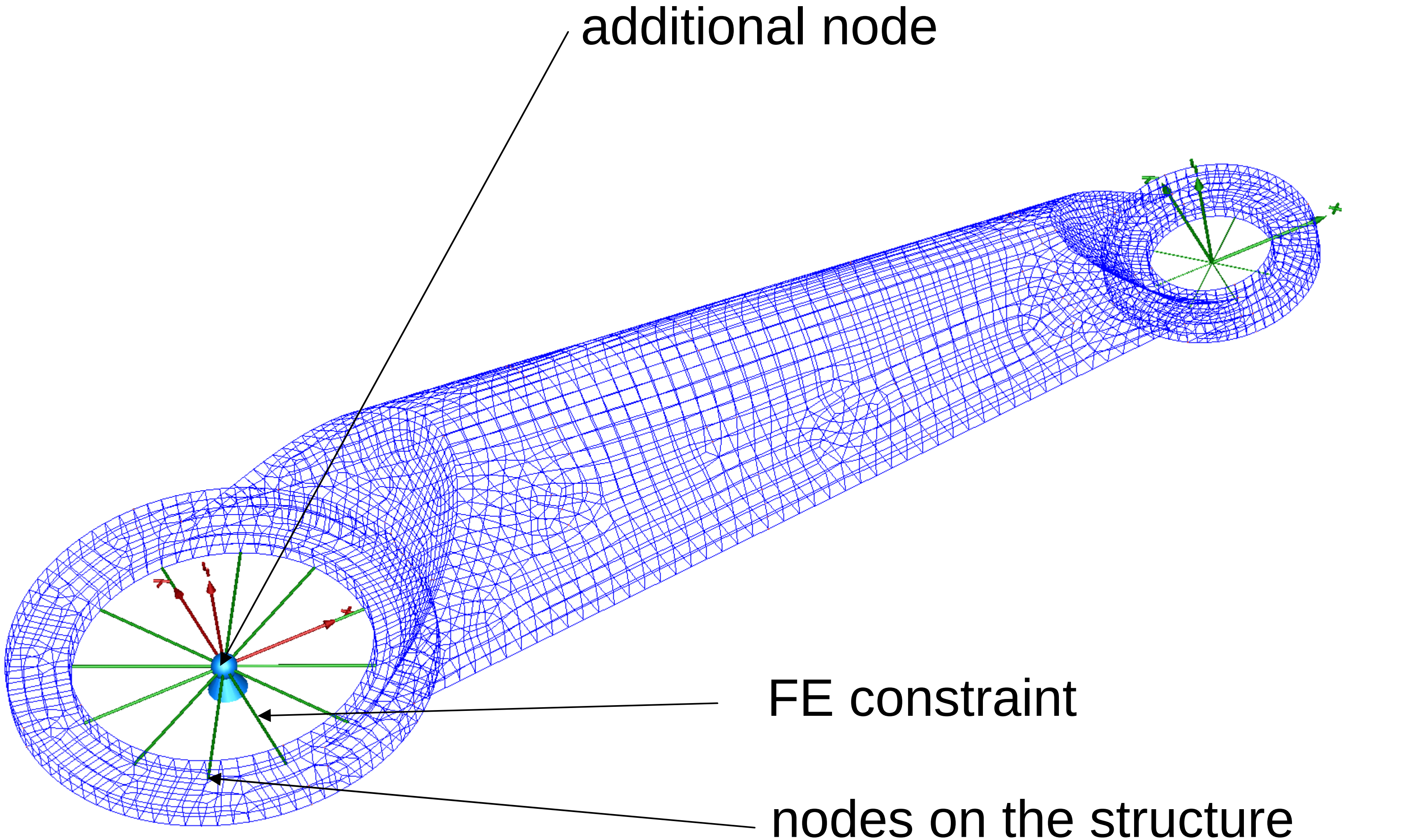
dynamic (natural) BC

$$EIw''(x_b) = 0$$

$$EIw'''(x_b) = 0$$

see e.g. Bremer92

Boundary Conditions II: FEM structures



Ritz-Approach (1908)

- fundamental idea: PDE, e.g.: $\Delta\Delta w = p(x,y)$

$$w_n = \sum_{i=1}^{n} a_i \psi_i(x) \qquad F = \int_a^b f(x, w_n, w_n', w_n'', ...)\,\mathrm{d}x \longrightarrow \text{stationary}$$

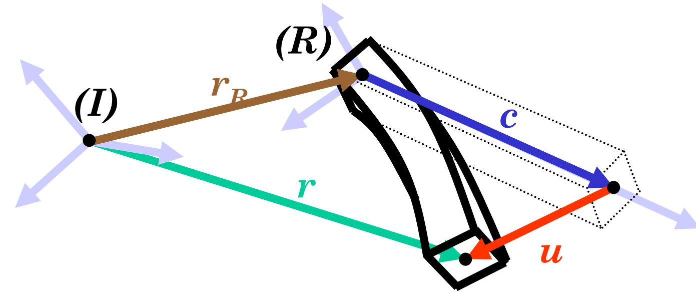
$$\boldsymbol{u} = \sum_{i=1}^{n} \boldsymbol{w}_i(x) q_i(t) \qquad \delta F = \delta \int_{t_0}^{t_1} (T - U)\,\mathrm{d}x = 0$$

- Equations of Motion $\rightsquigarrow$ optimisation problem $\Rightarrow$ weak solution
- provided that the chosen *n* mode functions
  - are linearly independent
  - the number *n* may be increased in order to improve the approximation quality
  - satisfy the kinematic boundary conditions

- for comparison: die strong solution satifies
  - the PDE + the kinematic + dynamic boundary conditions

- in practise: a good approximation of the dynamic boundary conditions improves the convergence of the weak solution (cp. (quasi-)comparison functions) see Meirovitch67

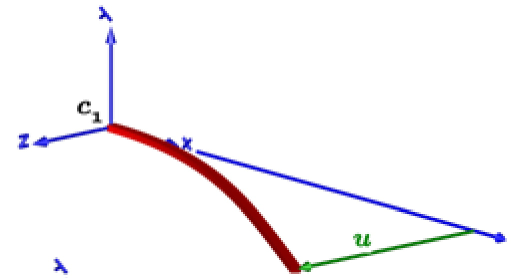
Reference Frames



$$\boldsymbol{r}(\boldsymbol{c},t) = \boldsymbol{r}_R(t) + \boldsymbol{c} + \boldsymbol{u}(\boldsymbol{c},t)$$

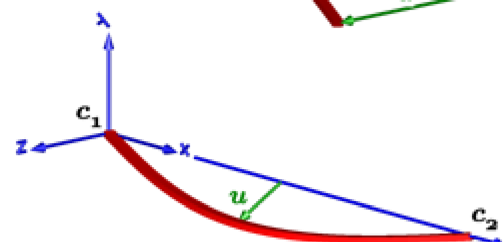
- 1st Option: tangent frame

$$\boldsymbol{u}(\boldsymbol{c}=0,t) = 0 \qquad \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{c}}(\boldsymbol{c}=0,t) = 0$$



- 2nd Option: chord frame

$$\boldsymbol{u}(\boldsymbol{c}_1) = 0 \qquad \boldsymbol{u}(\boldsymbol{c}_2)\cdot\overline{\boldsymbol{c}_1\boldsymbol{c}_2} = 0$$



- 3rd Option: Buckens frame

$$^0\boldsymbol{C}_r = {}^0\boldsymbol{C}_t = {}^1\boldsymbol{d}_C = \boldsymbol{O}$$

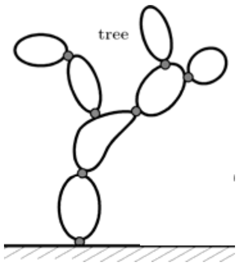
$$\begin{pmatrix} m\boldsymbol{I}_3 & & \text{sym.} \\ m\tilde{\boldsymbol{d}}_C & \boldsymbol{J} & \\ \boldsymbol{C}_t & \boldsymbol{C}_r & \boldsymbol{M}_e \end{pmatrix} \begin{pmatrix} \boldsymbol{a}_R \\ \boldsymbol{\alpha}_R \\ \ddot{\boldsymbol{q}} \end{pmatrix} = \boldsymbol{h}_\omega - \begin{pmatrix} 0 \\ 0 \\ \boldsymbol{K}_e\boldsymbol{q} + \boldsymbol{D}_e\dot{\boldsymbol{q}} \end{pmatrix} + \begin{pmatrix} \boldsymbol{f}_a \\ \boldsymbol{f}_\alpha \\ \boldsymbol{f}_e \end{pmatrix}$$

- Linearisation: choose reference frame in such a way that is as small as possible

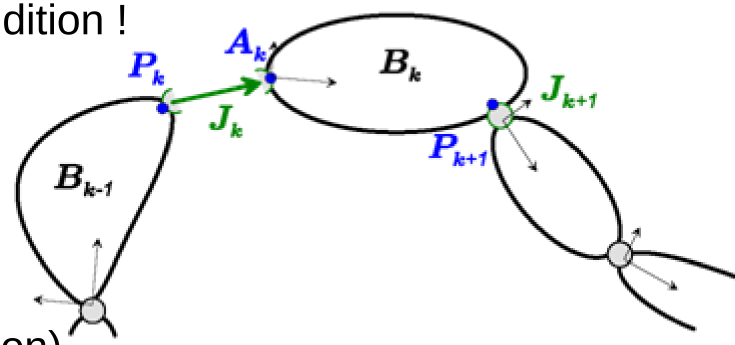
  $\boldsymbol{u} \ll 1 \quad \Rightarrow$ prefer Buckenssystem

see Schwertassek/Wallrapp/Shabana99

Proposal I: tree-structured Systems



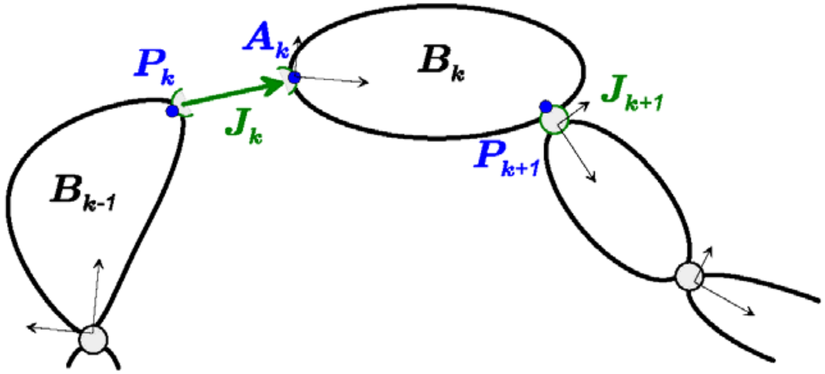
1. Kinematics of $\boldsymbol{P_k}$ only depends on generalised coordinates of preceding elements
2. A joint does not transmit a constraint force in the direction of its degree of freedom (dof)
3. no constraint force ⇒ no (i.e. free) boundary condition !

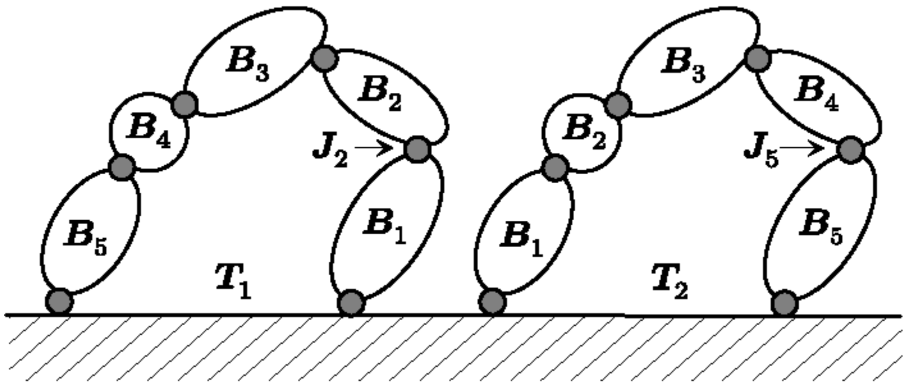


- boundary conditions at the joint $\boldsymbol{J_k}$ (in root direction)
  - choose according to the dof of the joint to which the flexible body is attached
- boundary condition at the joint $\boldsymbol{J_{k+1}}$ (to the next body of the tree)
  - the kinematic description does not rely on the existance of $\boldsymbol{B_{k+1}}$
  - choose free boundary conditions

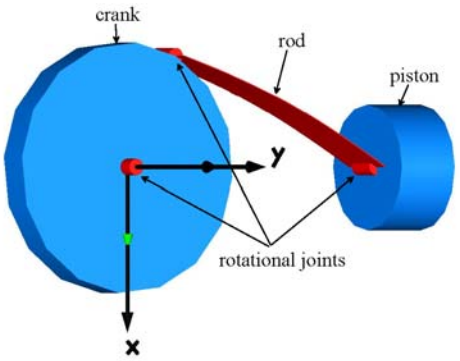
Proposal II: closed loops

- here the kinematic formulation of $\boldsymbol{P_{k+1}}$ is not independent from the successing body $\boldsymbol{B_{k+1}}$
  - in particular it is not unique(different indices possible)
  - several paths to reach the root of the mutibody system
- choose the boundary conditions at all joints according to their respective degrees of freedom



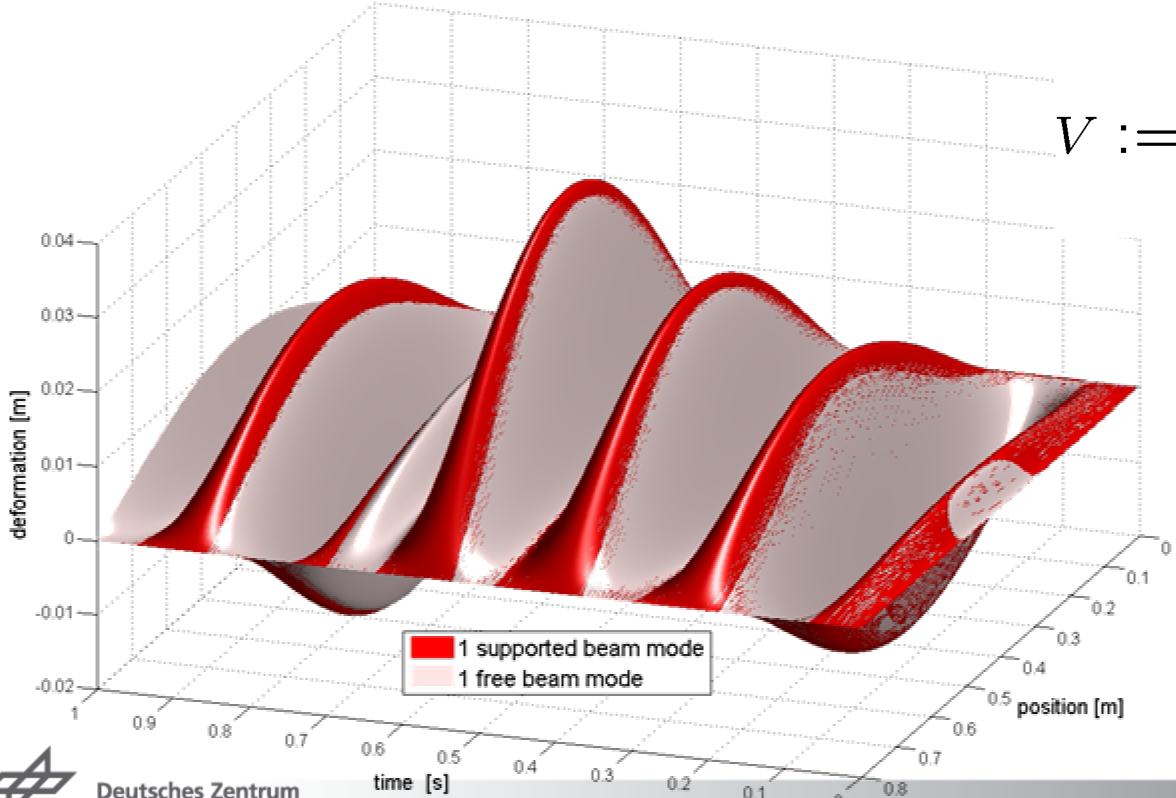


Slider Crank Example I



- Comparison
  - beam with supported-supported bcs
  - beam with free-free bcs (Shabana96)

$$V := \int_{t_0}^{t_1} \int_0^l ||\boldsymbol{u}_{CS} - \boldsymbol{u}_{BS}|| \, \mathrm{d}x \mathrm{d}t$$



Slider Crank Example II

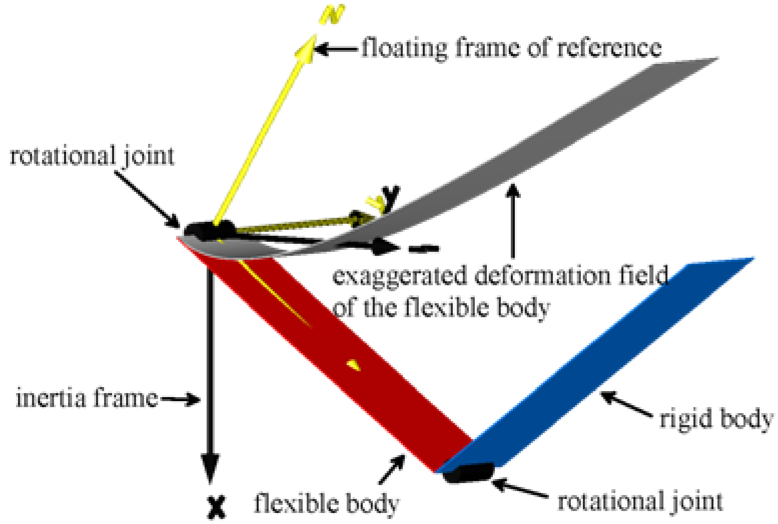
- for 5 % error V= 5.5
- Excitation is subcritical here: 1st eigenmode should dominate solution
- „supported modes“ are more efficient

Comparison A	n free modes versus n supported modes						
number of modes n	1	2	3	4	5	6	7
V $[10^{-4}\mathrm{m}^2\ \mathrm{s}]$	10.5	10.6	2.59	2.59	1.26	1.26	0.87

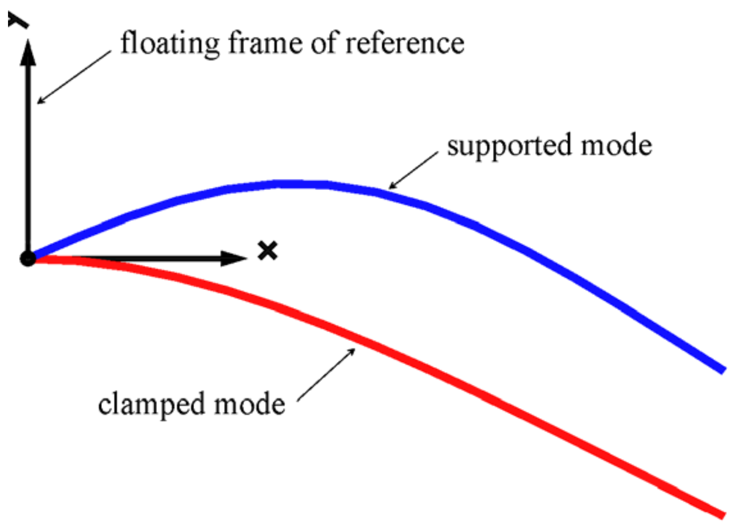
Comparison B	1 free mode versus n supported modes						
number of modes n	1	2	3	4	5	6	7
V $[10^{-4}\mathrm{m}^2\ \mathrm{s}]$	10.5	10.7	10.7	10.7	10.7	10.7	10.7

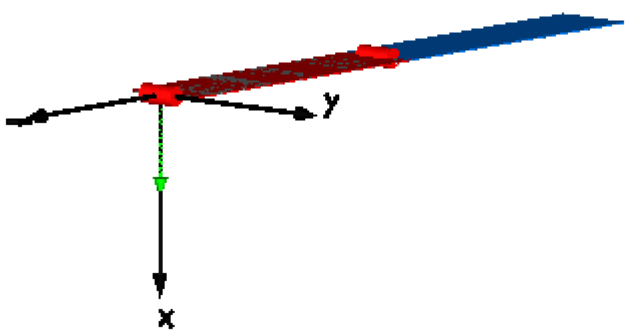
Comparison C	n free modes versus 1 supported mode						
number of modes n	1	2	3	4	5	6	7
V $[10^{-4}\mathrm{m}^2\ \mathrm{s}]$	10.5	10.7	3.00	3.07	2.05	2.06	1.83

Double Pendulum Example I



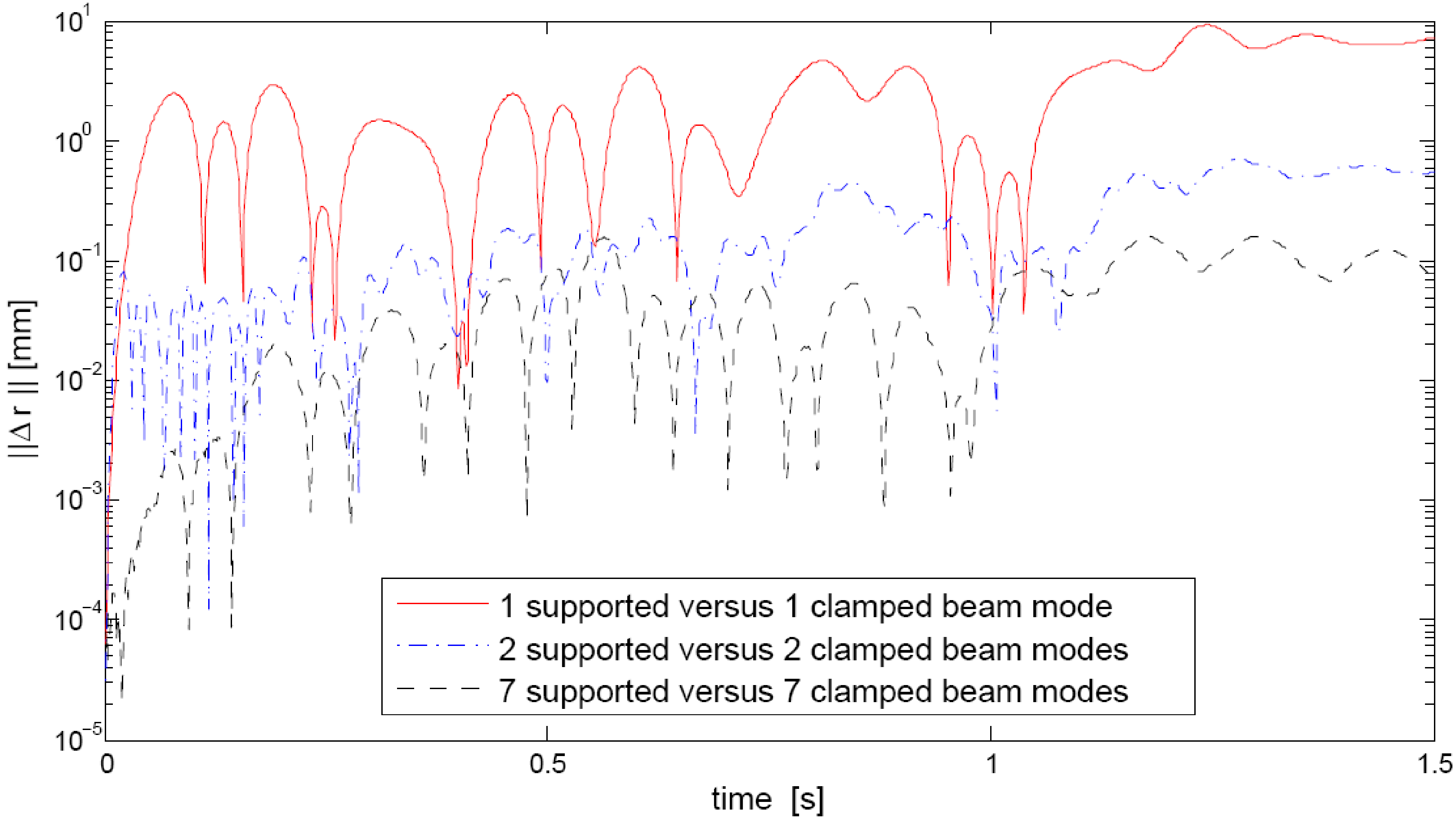
- Comparison:
  - supported modes
  - clamped modes



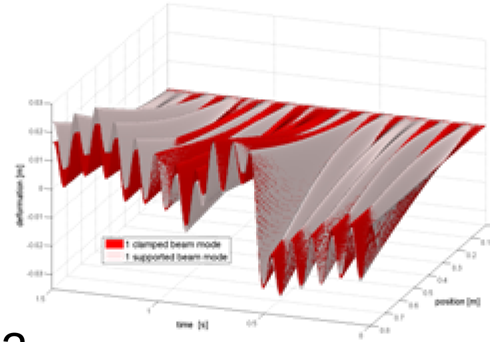


Double Pendulum Example II

- Displacements at end point of flexible body



Double Pendulum Example III



- Convergence of the solutions: 5% error here V= 2.62
- „supported mode“ is the more efficient approach

Comparison A	n clamped modes versus n supported modes						
number of modes n	1	2	3	4	5	6	7
V $[10^{-4}\mathrm{m}^2\ \mathrm{s}]$	21.9	1.85	0.56	0.43	0.42	0.42	0.46

Comparison B	1 clamped mode versus n supported modes						
number of modes n	1	2	3	4	5	6	7
V $[10^{-4}\mathrm{m}^2\ \mathrm{s}]$	21.9	23.8	24.3	24.5	24.6	24.6	24.6

Comparison C	n clamped modes versus 1 supported mode						
number of modes n	1	2	3	4	5	6	7
V $[10^{-4}\mathrm{m}^2\ \mathrm{s}]$	21.9	3.11	5.24	5.96	6.18	6.28	6.34

Summary

- tree-structured systems:
  - joint in root-direction: bc according to dof of the joint
  - other joints free bcs
- closed loops
  - all joints according to their dof
- see Multibody System Dynamics paper: On the Choice of Boundary Conditions for Mode Shapes in Flexible Multibody Systems