ECCOMAS Thematic Conference, Multibody Dynamics 2005
Universidad Politecnica de Madrid

Flexible Bodies with Thermoelastic Properties in Multibody Dynamics

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Theory: Constitutional and Field Equations

- **Material Constitution:**

mechanical field: stress, strain

thermal field: entropy density, temperature

$$\begin{pmatrix} \boldsymbol{\sigma} \\ \eta \end{pmatrix} = \begin{pmatrix} \boldsymbol{H}_c & -\boldsymbol{H}_\lambda^T \\ \boldsymbol{H}_\lambda & \boldsymbol{H}_a \end{pmatrix} \begin{pmatrix} \boldsymbol{\varepsilon} \\ \vartheta \end{pmatrix}$$

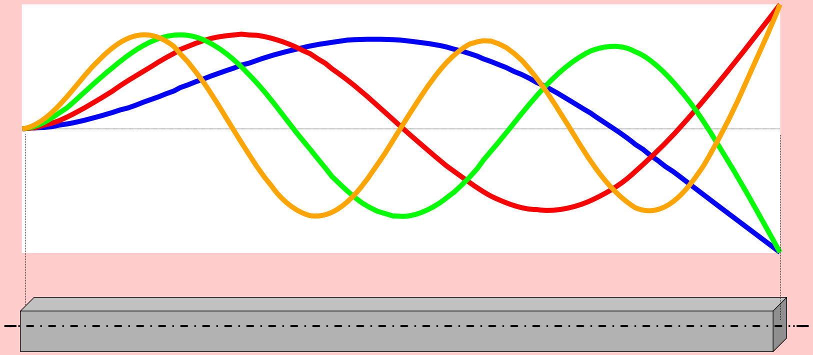
- **Weak Field Equations:**

$$\int \varrho \delta \boldsymbol{r}^T \ddot{\boldsymbol{r}} \, \mathrm{d}V + \int \delta \boldsymbol{\varepsilon}^T \underbrace{(\boldsymbol{H}_c \boldsymbol{\varepsilon} - \boldsymbol{H}_\lambda^T \vartheta)}_{\boldsymbol{\sigma}} \, \mathrm{d}V = \int \delta \boldsymbol{r}^T \boldsymbol{f} \, \mathrm{d}V$$

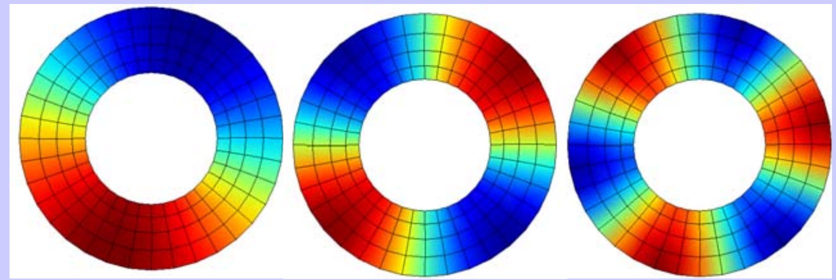
$$\int (\nabla \delta \vartheta)^T \boldsymbol{q} \, \mathrm{d}V - \int \delta \vartheta \underbrace{(\boldsymbol{H}_\lambda \dot{\boldsymbol{\varepsilon}} + \boldsymbol{H}_a \dot{\vartheta})}_{\dot{\eta}} \, \mathrm{d}V = \oint \delta \vartheta \boldsymbol{q}^T \boldsymbol{n} \, \mathrm{d}B$$

Theory: Modal Multifield Approach

Displacement Modes: $\boldsymbol{\Phi}_u$



Thermal Modes: $\boldsymbol{\Phi}_\vartheta$



$$\boldsymbol{u}(\boldsymbol{c},t) = \boldsymbol{\Phi}_u(\boldsymbol{c})\,\boldsymbol{z}_u(t) \qquad \vartheta = \boxed{\boldsymbol{\Phi}_\vartheta}\,\boldsymbol{z}_\vartheta$$

$$\varepsilon = (\nabla_u \boldsymbol{\Phi}_u)\,\boldsymbol{z}_u = \boxed{\boldsymbol{B}_u}\,\boldsymbol{z}_u$$

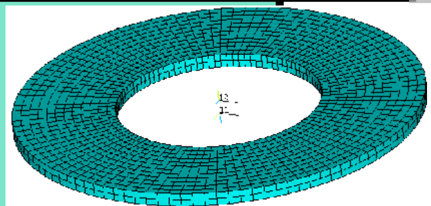
$$\boldsymbol{K}_{uu} = \int \boxed{\boldsymbol{B}_u^T}\,\boldsymbol{H}_c\,\boxed{\boldsymbol{B}_u}\,\mathrm{d}V \qquad \boldsymbol{K}_{u\vartheta} = \int \boxed{\boldsymbol{B}_u^T}\,\boldsymbol{H}_\lambda^T\,\boxed{\boldsymbol{\Phi}_\vartheta}\,\mathrm{d}V$$

$$\begin{pmatrix} \boldsymbol{M}_{aa} & \boldsymbol{M}_{a\alpha} & \boldsymbol{M}_{au} \\ & \boldsymbol{M}_{\alpha\alpha} & \boldsymbol{M}_{\alpha u} \\ \text{symm.} & & \boldsymbol{M}_{uu} \end{pmatrix} \begin{pmatrix} \boldsymbol{a}_R \\ \boldsymbol{\alpha}_R \\ \ddot{\boldsymbol{z}}_u \end{pmatrix} = \begin{pmatrix} \boldsymbol{h}_a \\ \boldsymbol{h}_\alpha \\ \boldsymbol{h}_u \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \boldsymbol{K}_{uu}\,\boldsymbol{z}_u - \boldsymbol{K}_{u\vartheta}\,\boldsymbol{z}_\vartheta \end{pmatrix}$$

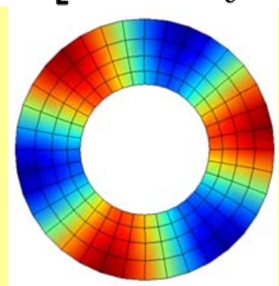
Theory: Thermal Response Modes

Modal Reduction

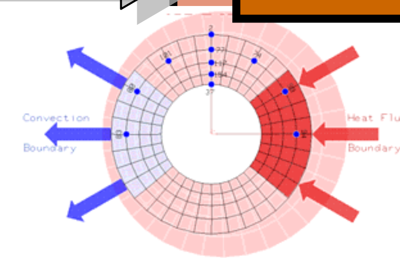
Heat Transfer Analysis



$$\Phi_\vartheta = [\dots \ \vartheta_i \ \dots]$$



Thermal Force Element



z_ϑ

Mechanical Analysis

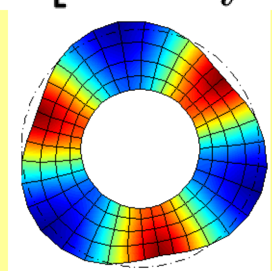
ϑ_i

$$f_i = f(\vartheta_i)$$

$$K_{u\vartheta} = [\dots \ f_i \ \dots]$$

$$u_i = K_{uu}^{-1} \cdot f_i$$

$$\Phi_u = [\dots \ u_i \ \dots]$$



Extended Equations of Motion

z_u

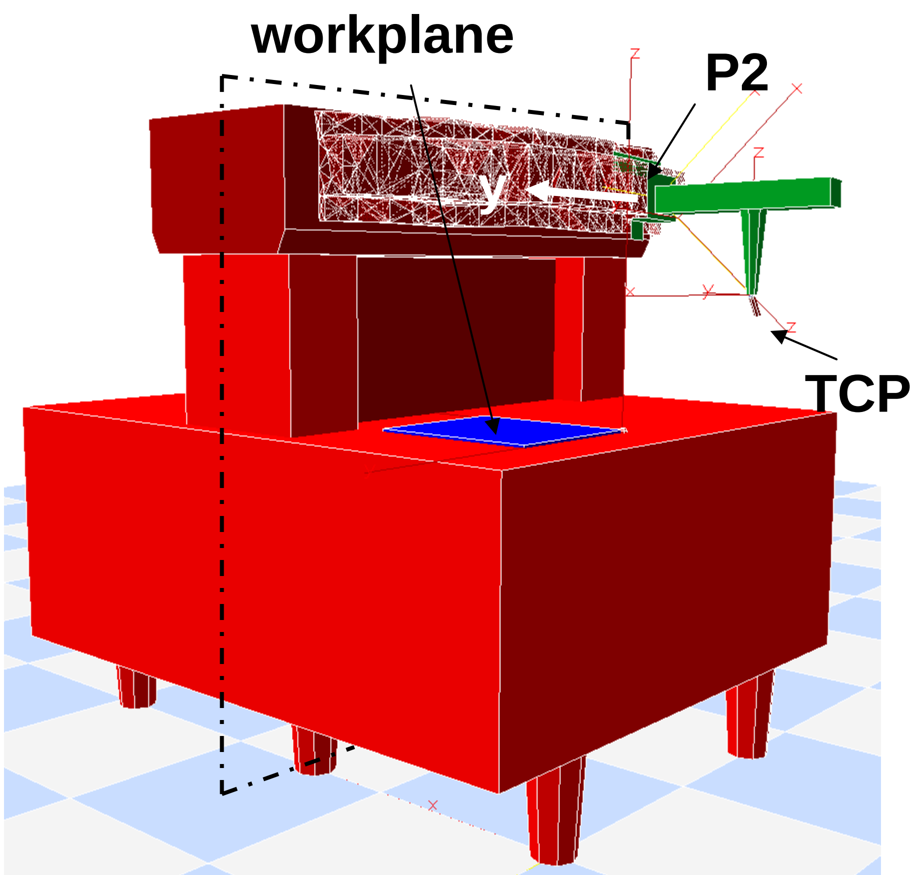
FEM

160 000 DoF

MBS

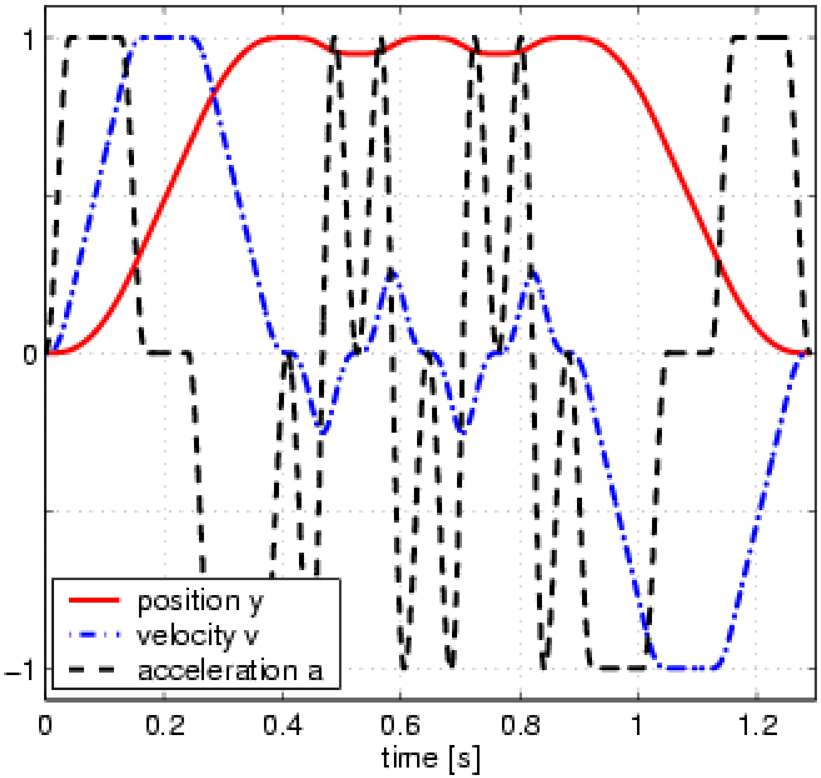
2 + DoF

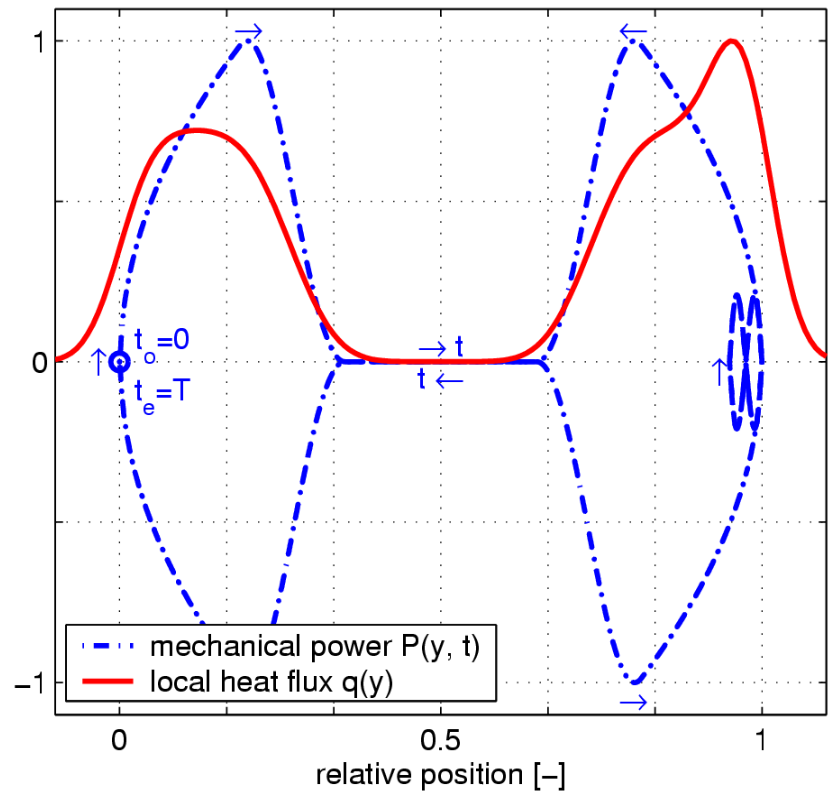
Machine Tool: The Name of the Game



- Point-to-Point working task
- Front and back side symmetric
- Drives with excellent dynamical properties for high performance !
- High accuracy demanded !
- Thermal loads caused by drives or working task
  - Relevant displacements ?
  - System-dynamical description ?
  - CAE work flow ?

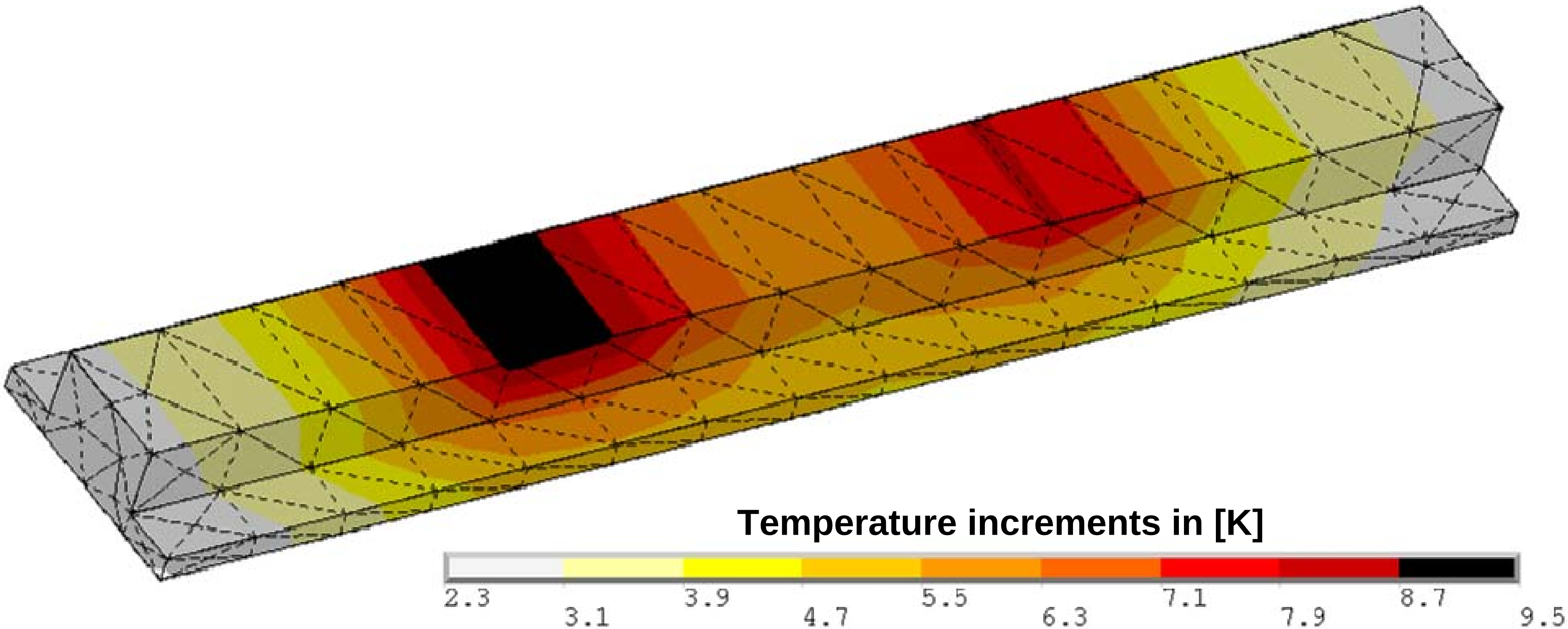
Machine Tool: Kinematic Scenario





$$\left.\begin{array}{rcl}\bar{q}(t) & = & \bar{q}(t+nT) \\ \bar{y}(t) & = & \bar{y}(t+nT) \\ n & \rightarrow & \infty\end{array}\right\} \Longrightarrow -\lambda\, \vartheta_{,yy} \approx q(y)$$

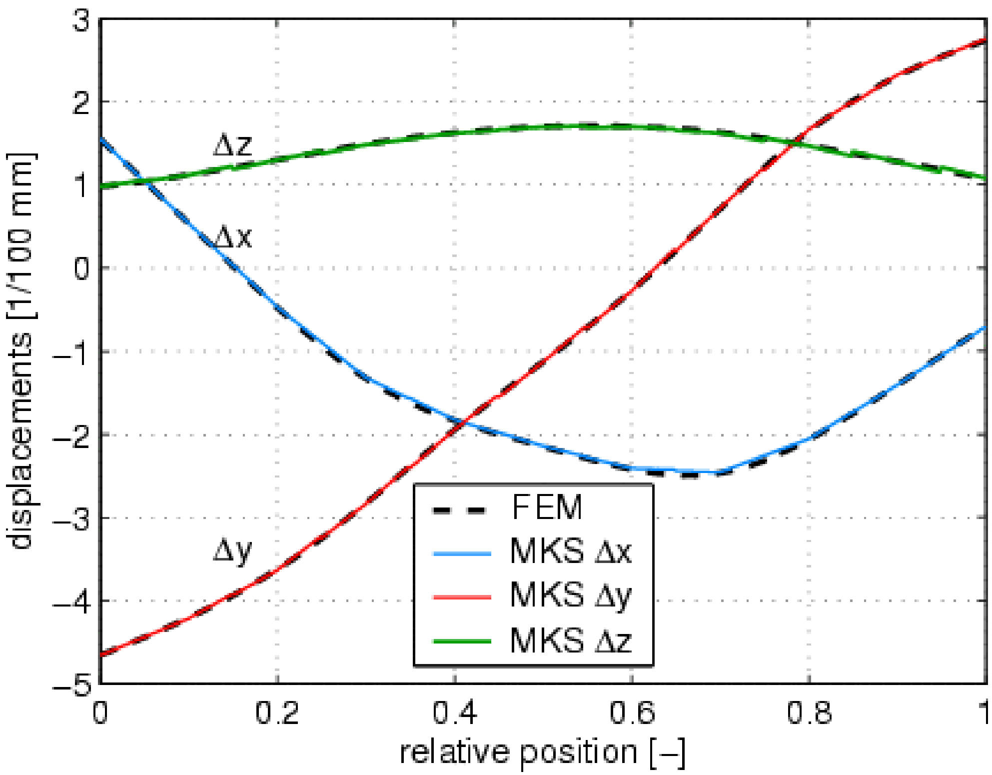
Machine Tool: Thermal Modes



2 steady state heat transfer FEM-solutions $\Rightarrow$ 2 thermal modes $\Rightarrow$ 2 thermal response modes

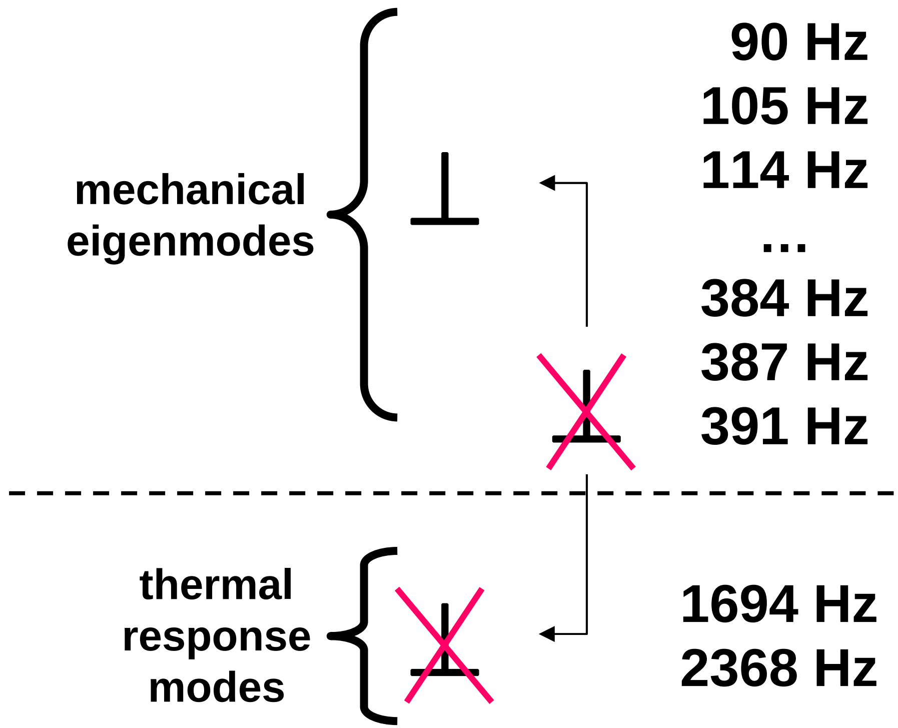
Machine Tool: Thermal Response Modes

Comparison FEM vs. MBS



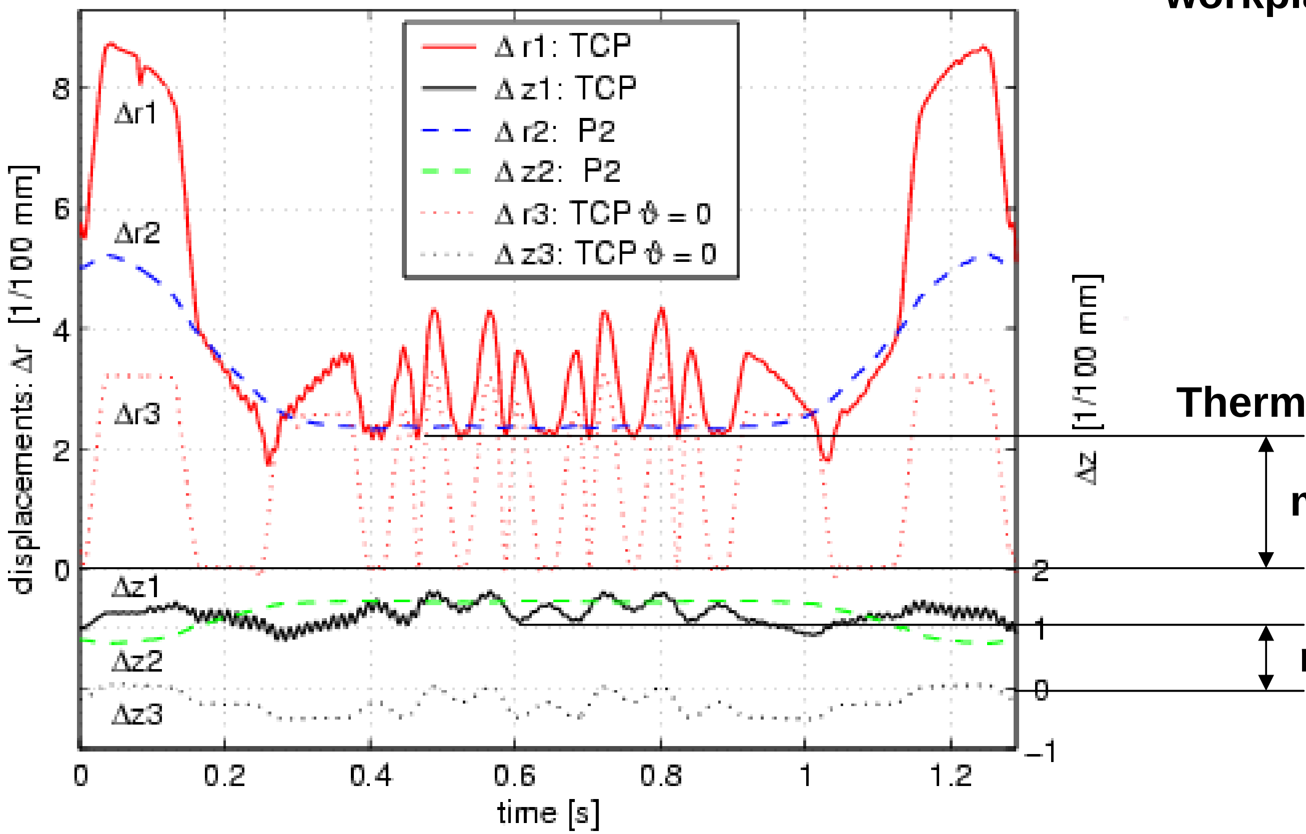
Thermal response modes
FEM : MBS ≈ 1:1

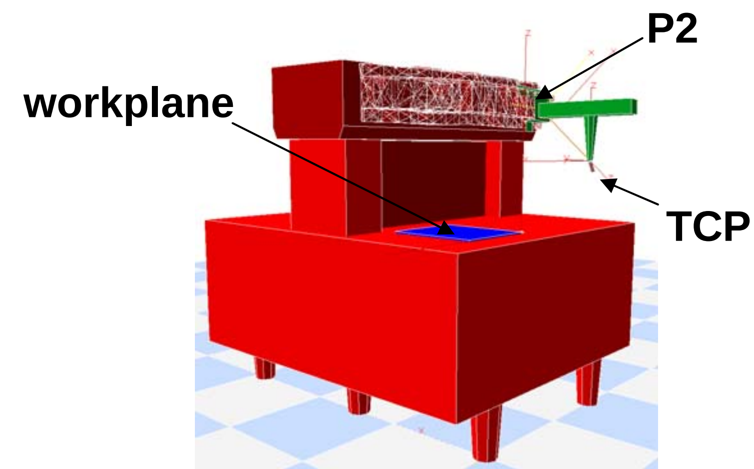
Eigenvalue Analysis



Eigenmodes and thermal response modes were weakly coupled.

Machine Tool: Dynamic Simulation





Thermal displacement at end position

measured in workplane

measured normal to workplane

TCP: tool center point

P2: reference point, moving along the machine base with predefined kinematics

Δr3, Δz3: reference displacements of TCP without thermal loads

Conclusions and Outlook

- Description and CAE workflow of thermal displacements is feasible.
- In general, modes are not orthogonal.
- High frequency modes are involved.

- Machine Tool:

  model extension regarding transient temperatures
- Methodology:

  neglect of inertia terms related to thermal response modes

Thank you for your attention