MANIPULABILITY AND ACCURACY MEASURES FOR A MEDICAL ROBOT IN MINIMALLY INVASIVE SURGERY

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Abstract This paper presents measures for manipulability and accuracy that are specifically adapted to the conditions found in robotically assisted minimally invasive surgery. The considered robot consists of 9 active joints, thus allowing for full manipulability at the tool tip as well as null-space motion. The presented manipulability and positioning accuracy measures are based on an "inverse Jacobian" approach since the constraints at the entry point into the human body forbid a classic formulation. High significance of the measures is reached by including robot design parameters such as encoder resolution and maximum joint velocity.

Keywords: manipulability, positioning accuracy, robotic assistance, minimally invasive surgery, optimisation, robot design

1. Introduction

In minimally invasive surgery (MIS), the field of operation is reached through small incisions into the human body with the use of thin cylindrical instruments. In the case of robotic assistance, these instruments are tele-operated by systems such as the daVinci™ or ZEUS™ robot. The movement of the instrument is constrained in two directions at the entry point into the human body as shown in Fig. 1. Since the operating room is an unpredictable and overcrowded environment, it is desirable to have a medical robot with a redundant kinematic structure, thus permitting the robot to avoid collisions without changing the position and orientation of the tool tip (null-space motion).

The optimal positioning of both the robot base(s) and the entry point(s) into the human body are determined in a preoperational planning step, where different base positions and entry points are evaluated according to predefined optimisation criteria such as e.g. manipulability, accuracy, geometrical considerations, insensitiveness of the robot setup or overall size of the robot.

Among these, manipulability and positioning accuracy are of great interest due to the following reasons:

- Singular configurations which could cause system failures must be avoided.
- A certain minimal velocity of the instrument tip both in terms of translation and rotation must be realised all over the considered operating field. This is important to assure that motions commanded by the surgeon can be performed by the robot system.
- The demanded accuracy of the positioning of the instrument tip must be guaranteed. This is of great interest if very fine structures (e.g. blood vessels) are manipulated.

Previous work concerning preoperational planning of MIS procedures (Adhami, 2002; Chiu et al., 2000; Engel et al., 2003; Lehmann et al., 2001; Selha et al., 2003; Tabaie et al., 1999) does however not take into account manipulability and positioning accuracy sufficiently. Instead, the used approaches *tool dexterity* (Adhami, 2002; Lehmann et al., 2001) as well as *magic pyramid* (Chiu et al., 2000) evaluate the orientation of the instruments, the endoscope, and the surface normal of the considered operating field to each other. Engel et al., 2003 consider the distance of the joint angles of the robot from its maximum values. Experimental trials (Adhami, 2002) suggest the conclusion that the use of more sophisticated descriptions of manipulability and accuracy measures is reasonable. Since their classic formulations are not applicable to the MIS setup due to the restriction at the entry point, this paper presents a suitable formulation.

In the next section, the underlying problem is described. The derivation of measures for manipulability and accuracy is shown in Sect. 3 and 4. The results are discussed in Sect. 5 and a conclusion is given in Sect 6.

2. Problem Statement

A robot with 7 degrees of freedom (DoFs) as shown in Fig. 1 is considered. Two additional DoFs (Θ_8 and Θ_9) are added by the actuated instrument. The notation Θ_i stands both for the joint i and the axis direction parallel to the rotation axis of that same joint. Taking into account the loss of 2 DoFs due to the kinematic restriction at the entry

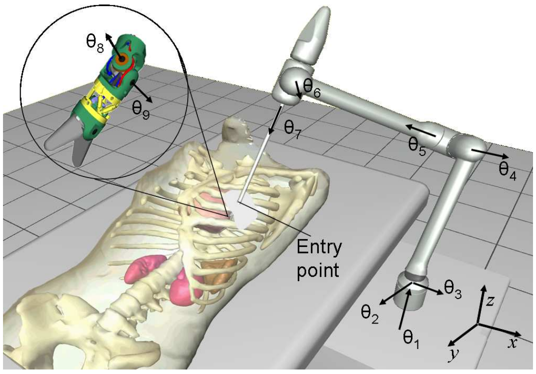

Figure 1. Surgical robot with actuated instrument.

point into the human body, full 6 DoFs remain for the manipulation of the instrument tip and an extra degree of freedom serves to accomplish null-space motion.

Dynamics are not included in the modelling since typical motions in MIS are too small to induce significant forces.

3. Manipulability

The classic formulation of manipulability based on YOSHIKAWA, 1990 investigates the singular values of the Jacobian matrix $\mathbf{P} \in \mathbb{R}^{6\times 9}$ in Eq. 1:

$$\mathbf{v} = \mathbf{P}\dot{\mathbf{\Theta}}\,, \tag{1}$$

$$\text{with} \quad \dot{\mathbf{\Theta}} = [\dot{\Theta}_1,\ ...,\ \dot{\Theta}_9]^{\mathrm{T}} \quad \text{the joint velocities} \tag{2}$$

$$\text{and} \quad \mathbf{v} = [v_x,\ v_y,\ v_z,\ \omega_x,\ \omega_y,\ \omega_z]^{\mathrm{T}} \quad \text{the translational velocity} \tag{3}$$

of the instrument tip and the angular velocity of the last segment of the instrument expressed in the inertial coordinate system $[x, y, z]$ as depicted in Fig. 1. However, in the considered case Eq. 1 cannot be used to address manipulability because it does not take into account the motion restriction due to the entry and the possibility of null-space motion. At the entry point, the following kinematical constraint must be satisfied (Ortmaier and Hirzinger, 2000):

$$\mathbf{C}_1\dot{\mathbf{\Theta}} = \mathbf{0}\,, \mathbf{C}_1 \in \mathbb{R}^{2\times 9}. \tag{4}$$

Null-space motion implies that a given tool tip position and orientation can be realised with an infinite number of joint sets. To calculate the most appropriate joint set, another constraint has to be introduced which represents some kind of optimisation among the possible joint sets:

$$\mathbf{C}_2\dot{\boldsymbol{\Theta}} = \mathbf{0}\,, \mathbf{C}_2 \in \mathbb{R}^{1\times 9}, \tag{5}$$

resulting in

$$\mathbf{C}\dot{\boldsymbol{\Theta}} = \mathbf{0} \quad \text{with} \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \end{bmatrix} \in \mathbb{R}^{3\times 9}. \tag{6}$$

If the vector of joint velocities $\dot{\boldsymbol{\Theta}}$ is divided arbitrarily into a dependent part $\dot{\boldsymbol{\Theta}}_{\mathrm{d}} \in \mathbb{R}^{3\times 1}$ and an independent part $\dot{\boldsymbol{\Theta}}_{\mathrm{i}} \in \mathbb{R}^{6\times 1}$, it can be reordered as follows:

$$\dot{\boldsymbol{\Theta}} = \begin{bmatrix} \dot{\boldsymbol{\Theta}}_{\mathrm{d}} \\ \dot{\boldsymbol{\Theta}}_{\mathrm{i}} \end{bmatrix}. \tag{7}$$

The equations 1 and 6 can then be rewritten as (Ortmaier and Hirzinger, 2000):

$$\mathbf{v} = \mathbf{P}_{\mathrm{d}}\dot{\boldsymbol{\Theta}}_{\mathrm{d}} + \mathbf{P}_{\mathrm{i}}\dot{\boldsymbol{\Theta}}_{\mathrm{i}} \quad \text{with} \quad \mathbf{P} = [\mathbf{P}_{\mathrm{d}} \;\; \mathbf{P}_{\mathrm{i}}] \tag{8}$$

$$\text{and} \quad \dot{\boldsymbol{\Theta}}_{\mathrm{d}} = \mathbf{B}\dot{\boldsymbol{\Theta}}_{\mathrm{i}} \quad \text{with} \quad \mathbf{B} \in \mathbb{R}^{3\times 6}. \tag{9}$$

After insertion of the kinematical constraint (Eq. 6), Eq. 8 has form

$$\mathbf{v} = \mathbf{P}'\dot{\boldsymbol{\Theta}}_{\mathrm{i}} \quad \text{with} \quad \mathbf{P}' = \mathbf{P}_{\mathrm{d}}\mathbf{B} + \mathbf{P}_{\mathrm{i}} \in \mathbb{R}^{6\times 6}, \tag{10}$$

where only 6 of the 9 joint velocities occur. Depending on which joint velocities are chosen as dependent, the Jacobian matrix $\mathbf{P}'$ has different elements and different singular values. Therefore, it is not useful for the formulation of manipulability. A more suitable formulation is achieved if the inverse correlation is used. This can be done by solving Eq. 10 for $\dot{\boldsymbol{\Theta}}_{\mathrm{i}}$ and combining the result with Eq. 9 (Konietschke, 2001):

$$\dot{\boldsymbol{\Theta}} = \mathbf{D}\mathbf{v} \quad \text{with} \quad \mathbf{D} = \begin{bmatrix} \mathbf{B}\mathbf{P}'^{-1} \\ \mathbf{P}'^{-1} \end{bmatrix}. \tag{11}$$

This equation includes all joint velocities. It relates a given instrument velocity $\mathbf{v}$ to the joint velocities $\dot{\boldsymbol{\Theta}}$ in a non-ambiguous way for every non-singular robot configuration by consideration of the kinematic constraint due to the entry point. Alternatively to the above described calculations, matrix $\mathbf{D}$ can also be obtained by numerical differentiation. This can be done by applying difference quotients to the inverse kinematics which expresses the joint angles as function of the instrument tip position and orientation and the position of the entry point.

To define a measure for manipulability, a connection between the maximum joint velocity $\dot{\Theta}_{\max}$ and the desired minimal translational velocity $\dot{x}_{\min}$ and angular velocity $\dot{\alpha}_{\min}$ of the instrument tip has to be established. To do so, the matrix $\mathbf{D}$ in Eq. 11 is split into two components

$$\mathbf{D} = [\mathbf{E}\,\mathbf{F}] \quad \text{with} \quad \mathbf{E} = \begin{bmatrix} e_{11} & & e_{13} \\ \vdots & \ddots & \vdots \\ e_{81} & & e_{83} \end{bmatrix}, \mathbf{F} = \begin{bmatrix} f_{11} & & f_{13} \\ \vdots & \ddots & \vdots \\ f_{81} & & f_{83} \end{bmatrix}, \tag{12}$$

and Eq. 11 is rewritten:

$$\dot{\boldsymbol{\Theta}} = \mathbf{E} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} + \mathbf{F} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}. \tag{13}$$

For each joint $i \in [1, ..., 8]$, the maximum velocity $\dot{\Theta}_{i,\,\max}$ is computed by solving the optimisation problem (14):

$$\dot{\Theta}_{i,\max} = \left\{ |\dot{\Theta}_i(\dot{\mathbf{x}})| \xrightarrow{\text{opt}} \max \right\} = \left\{ |e_{i1}\dot{x} + e_{i2}\dot{y} + e_{i3}\dot{z} + f_{i1}\dot{\alpha}_1 + f_{i2}\dot{\alpha}_2 + f_{i3}\dot{\alpha}_3| \xrightarrow{\text{opt}} \max \right\} \tag{14}$$

under the constraints

$$\sqrt{v_x^2 + v_y^2 + v_z^2} - \dot{x}_{\min} \leq 0 \quad \text{and} \quad \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2} - \dot{\alpha}_{\min} \leq 0 \,. \tag{15}$$

Solution is done using the Lagrange function and yields (see Konietschke, 2001 for further details):

$$\dot{\Theta}_{i,\,\max} = \sqrt{e_{i1}^2 + e_{i2}^2 + e_{i3}^2}\;\dot{x}_{\min} + \sqrt{f_{i1}^2 + f_{i2}^2 + f_{i3}^2}\;\dot{\alpha}_{\min}\,. \tag{16}$$

Thus the maximum joint velocity $\dot{\Theta}_{\max} = \max(\dot{\Theta}_{i,\,\max})$ is determined which can be used as a reciprocal manipulability measure:

$$w_{\text{man}} = \frac{1}{\dot{\Theta}_{\max}}\,. \tag{17}$$

For a given, desired minimal velocity at the tool tip, low values of $\dot{\Theta}_{\max}$ denote that the joint velocities remain low. This indicates good manipulability.

4. Positioning accuracy

The following question is considered to define an (in)accuracy measure:

> How far can the instrument tip be moved at maximum with all the changes in the articular space remaining below the encoder resolution $\Delta\Theta_{\text{min}}$?

This describes the maximum translational and rotational movement Δx_{min} and $\Delta\alpha_{\text{min}}$ of the instrument tip that is not detectable by the robot control system and thus provides a worst case estimation of the positioning accuracy that can be commanded.

For small changes $\mathbf{\Delta\Theta}$ and $\mathbf{u}$, where $\mathbf{u} = \begin{bmatrix} \mathbf{u}_{\text{trans}} \\ \mathbf{u}_{\text{rot}} \end{bmatrix}$ describes small displacements of the instrument tip and small changes in the orientation of the instrument, the following relation approximately holds:

$$\Delta\mathbf{\Theta} = \mathbf{D}\mathbf{u}\,. \tag{18}$$

Since the vector $\mathbf{u}$ contains both translational and rotational components, normalisation is applied:

$$\Delta\mathbf{\Theta} = \tilde{\mathbf{D}}\tilde{\mathbf{u}} \quad \text{with} \quad \tilde{\mathbf{u}} = \begin{bmatrix} \mathbf{u}_{\text{trans}}/\Delta x_{\text{min}} \\ \mathbf{u}_{\text{rot}}/\Delta\alpha_{\text{min}} \end{bmatrix}, \ \tilde{\mathbf{D}} = [\Delta x_{\text{min}}\mathbf{E} \ \ \Delta\alpha_{\text{min}}\mathbf{F}]\,. \tag{19}$$

Thus, calculating the smallest singular value $\tilde{\sigma}_{\text{min}}$ of $\tilde{\mathbf{D}}$ yields (Konietschke, 2001):

$$\|\Delta\mathbf{\Theta}\|_2 \geq \tilde{\sigma}_{\text{min}} \|\tilde{\mathbf{u}}\|_2\,. \tag{20}$$

One is interested in the maximum change of one of the joint angles which is synonymous to the maximum norm $\|\Delta\mathbf{\Theta}\|_\infty$ and not to the Euclidean norm as appearing in Eq 20. Therefore, the following estimation is used:

$$\|\Delta\mathbf{\Theta}\|_2 \geq \|\Delta\mathbf{\Theta}\|_\infty \cdot \sqrt{\dim(\Delta\mathbf{\Theta})}\,. \tag{21}$$

With movements $\|\mathbf{u}_{\text{trans}}\|_2 \leq \Delta x_{\text{min}}$ and $\|\mathbf{u}_{\text{rot}}\|_2 \leq \Delta\alpha_{\text{min}}$, Eq. 22 holds:

$$\|\tilde{\mathbf{u}}\|_2 = \sqrt{\|\mathbf{u}_{\text{trans}}\|_2^2/\Delta x_{\text{min}}^2 + \|\mathbf{u}_{\text{rot}}\|_2^2/\Delta\alpha_{\text{min}}^2} \leq \sqrt{2}\,. \tag{22}$$

Thus, the following measure w_{pos} for positioning accuracy can be established:

$$w_{\text{pos}} = \Delta\Theta_{\text{min}} = \frac{\tilde{\sigma}_{\text{min}}}{\sqrt{\frac{1}{2}\dim(\Delta\Theta)}}\,, \tag{23}$$

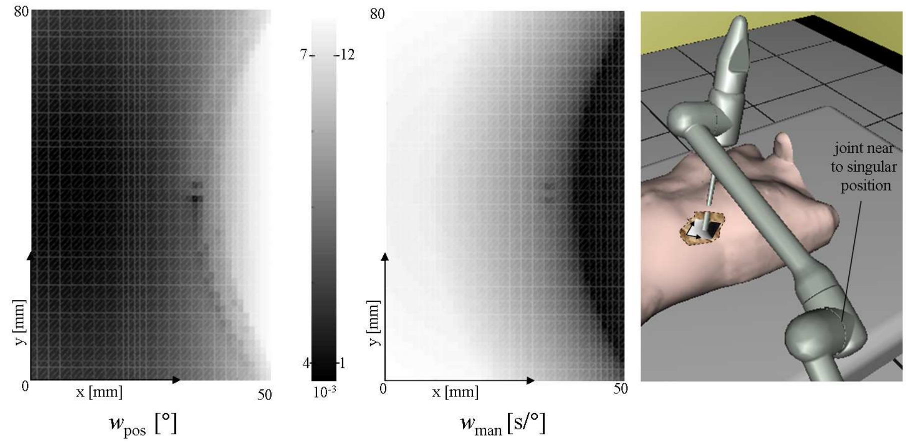

Figure 2. The measures w_{man} and w_{pos} in the vicinity of a singular configuration.

where $\Delta\Theta_{\mathrm{min}}$ is the necessary encoder resolution to detect movements greater than Δx_{min} resp. $\Delta\alpha_{\mathrm{min}}$.

5. Results

Fig. 2 shows the manipulability measure w_{man} and the positioning accuracy w_{pos} for different instrument tip positions in the vicinity of a singularity. Since w_{man} decreases when approaching the border line to singularity, it can be used to omit singular configurations.

The measure for positioning accuracy w_{pos}mainly shows a reverse behavior compared to w_{man} what is understandable since high manipulability is defined to allow fast motion of the tool tip with low joint velocities whereas high positioning accuracy means small displacements with large joint rotations. Concerning the validation of a robot configuration, it has to be found a trade off between high manipulability and high positioning accuracy.

6. Conclusion

In this paper, modified measures for manipulability and accuracy are presented to overcome the problems such as unexpected singularities or poor manipulability that were encountered with previous approaches in the field of robotically assisted MIS. The presented measures are very descriptive since they comprise the robotic design parameters *encoder resolution* and *maximum joint velocity.* In order to establish quantitative information whether a certain value for w_{man} or w_{pos} is sufficient to accomplish robotically assisted MIS, the requirements concerning desired velocity and positioning accuracy of the tool tip have to be known. In this context, work remains to be done since previous publications (Riviere and Jensen, 2000; Singh and Riviere, 2002; Hotraphinyo and Riviere, 2001; Cao et al., 1996) only cover analyses of translational aspects.

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