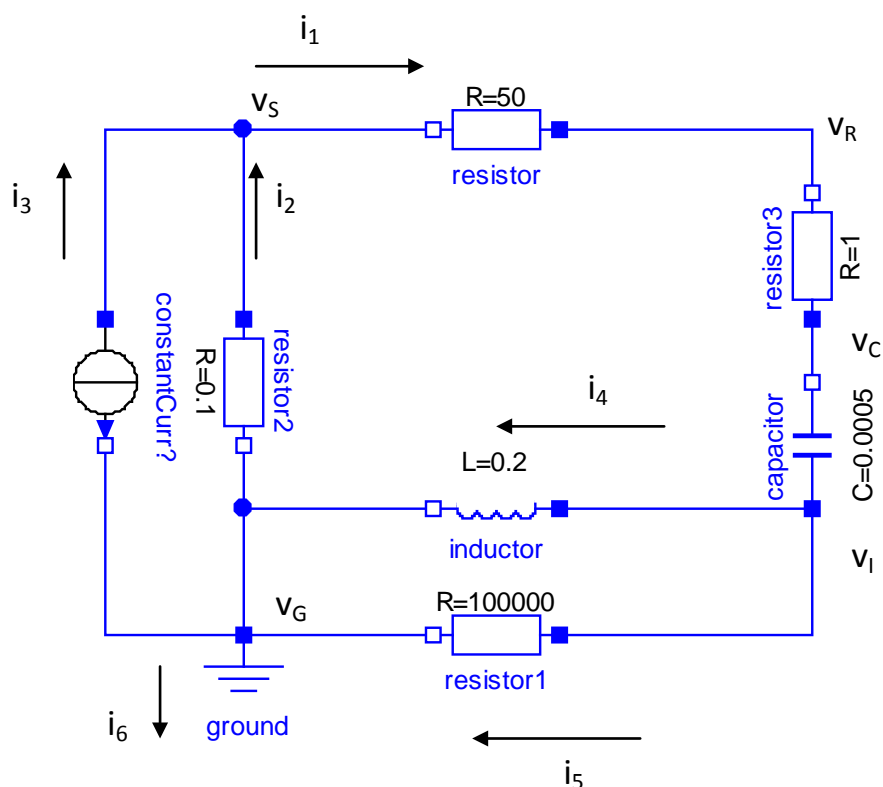Dr. Dirk Zimmer

Virtual Physics

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Exercise 5: Tearing Algebraic Loops within an electric circuit

Solution

Task A: Set up the model equations

We select the 6 currents and 5 voltage potentials as selected above. Additionally, we introduce uC as voltage across the capacitor. This yields 12 variables in total.

There are 3 node equations for the corrent:

i1 = i2+i3

i4+i5 = i1

i4+i5+i6 = i2+i3

The remaining 9 equations result from the components.

$i3 = I0$

$vS - vG = i2*R2$

$vR - vS = i1*R0$

$vC - vR = i1*R3$

$vI-vC = uC$

$der(uC)*C = i1;$

$der(i4)*I = vI-vG$

$vG-vI = R1*i5$

$vG = 0;$

Task B: Identify suitable tearing variables

uC and i4 are potential state variables and can assumed to be known.

Still only one two equations can be causalized:

$i3 := I0;$

$vG := 0;$

To continue, we select a tearing variable. i1 seems to be a good choice. It occurs in many equations and causalizes plenty of them.

i1 := iteration variable

$i2 := i1 - i3;$

$vS := i2*R2 + vG$

$vR := i1*R0 + vS$

$vC := i1*R3 + vR$

$vI := vC + uC;$

$i5 := i1 - i4;$

residual := vG-vI - R1*i5

der(uC) := i1/C;

der(i4) := (vI – vG)/I

i6 := i2 + i3 - i4 -i5

Remark: the equations der(i4) := (vI – vG)/I and der(uC) := i1/C are not part of the algebraic loop since the are not required for the computation of the residual.

This is the corresponding BLT form of the system:

i3	vG	i2	vS	vR	vC	vI	i5	i1	duC/dt	di4/dt	i6
X											
	X										
X		X						X			
	X		X					X			
			X	X				X			
				X	X			X			
					X	X					
							X	X			
	X					X	X				
								X	X		
	X					X				X	
X		X						X			X

Residual equations may lead to empty elements on the diagonal.