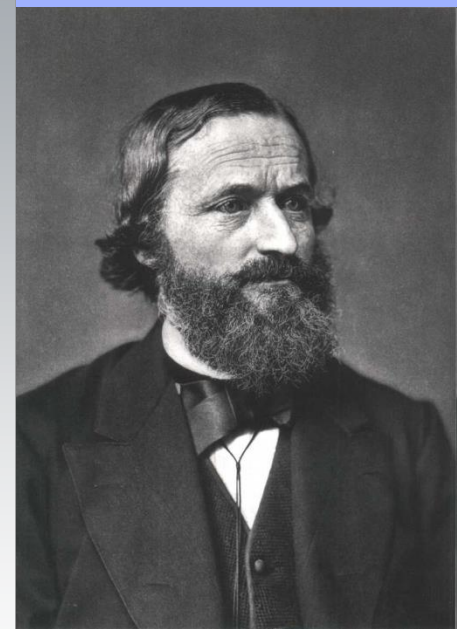


# Virtual Physics

## Equation-Based Modeling

TUM, November 08, 2022

Object-oriented formulation of physical systems – Part II

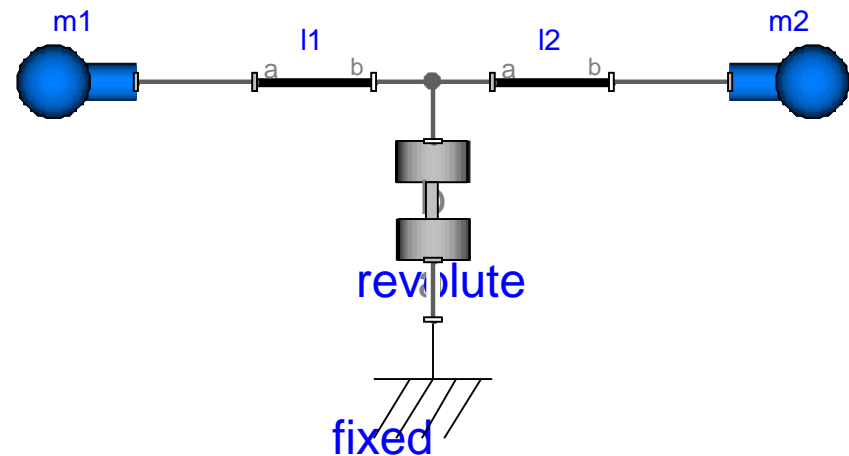
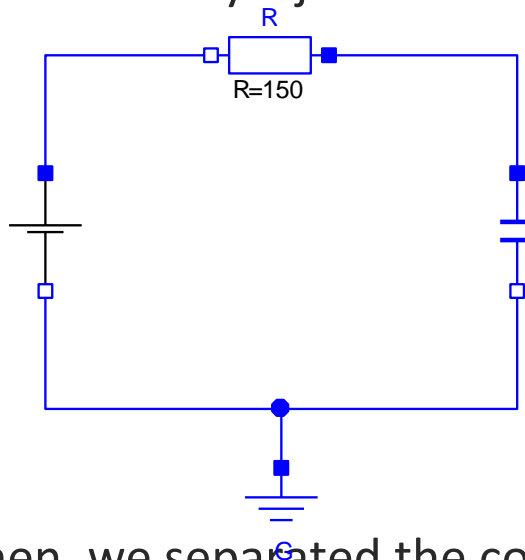


Dr. Dirk Zimmer

German Aerospace Center (DLR), Robotics and Mechatronics Centre

Attentive students may have noticed that we have done the same thing twice in the last hour.

- For mechanic or electric systems, the procedure was actually the same.
- First we decomposed the system into different components that are connected by a junction structure.



- Then, we separated the component equations from the connection equations.

- For each node in the junction structure, we defined a set of equations.
- Each node was represented by a pair of variables

A **potential** variable

$v$  (voltage potential for electrics)

$v$  (velocity for mechanics)

and a **flow** variable

$i$  (current for electrics)

$f$  (force for mechanics)

- For one connection between a set of  $n$  nodes,  $n$  equations have to be generated.

- **$n-1$  equalities**

In electrics:  $v_1 = v_2 = \dots = v_n$  (Kirchhoff's 2<sup>nd</sup> law)

In mechanics:  $v_1 = v_2 = \dots = v_n$  (Rigid constraint equation)

- **1 balance equation**

In electrics:  $i_1 + i_2 + \dots + i_n = 0$  (Kirchhoff's 1<sup>st</sup> law)

In mechanics:  $f_1 + f_2 + \dots + f_n = 0$  (D'Alembert's Principle)

But there is more to it:

- What does the product of the **mechanic** pair of connector variables represent?

$$v \text{ [m/s]} \cdot f \text{ [N]} = p \text{ [Nm/s]}$$

**It represents a flow of energy!** [Nm] is work/energy

- What does the product of the **electric** pair of connector variables represent?

$$v \text{ [Nm/C]} \cdot I \text{ [C/s]} = p \text{ [Nm/s]}$$

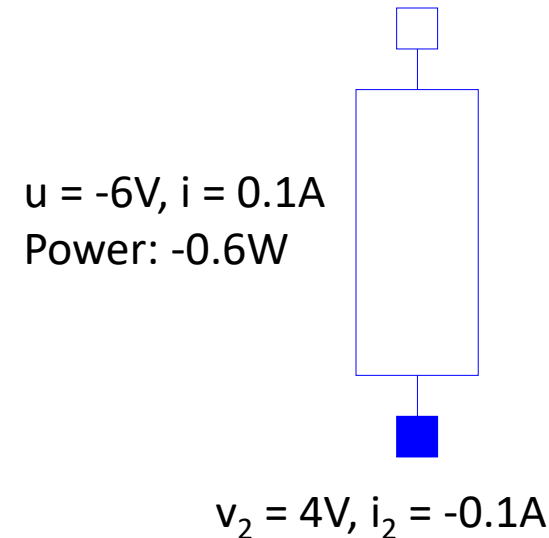
**It represents a flow of energy too!**

**This is not a coincidence! It indicates a general physical principle!**

Each component exhibits a certain behavior w.r.t. energy

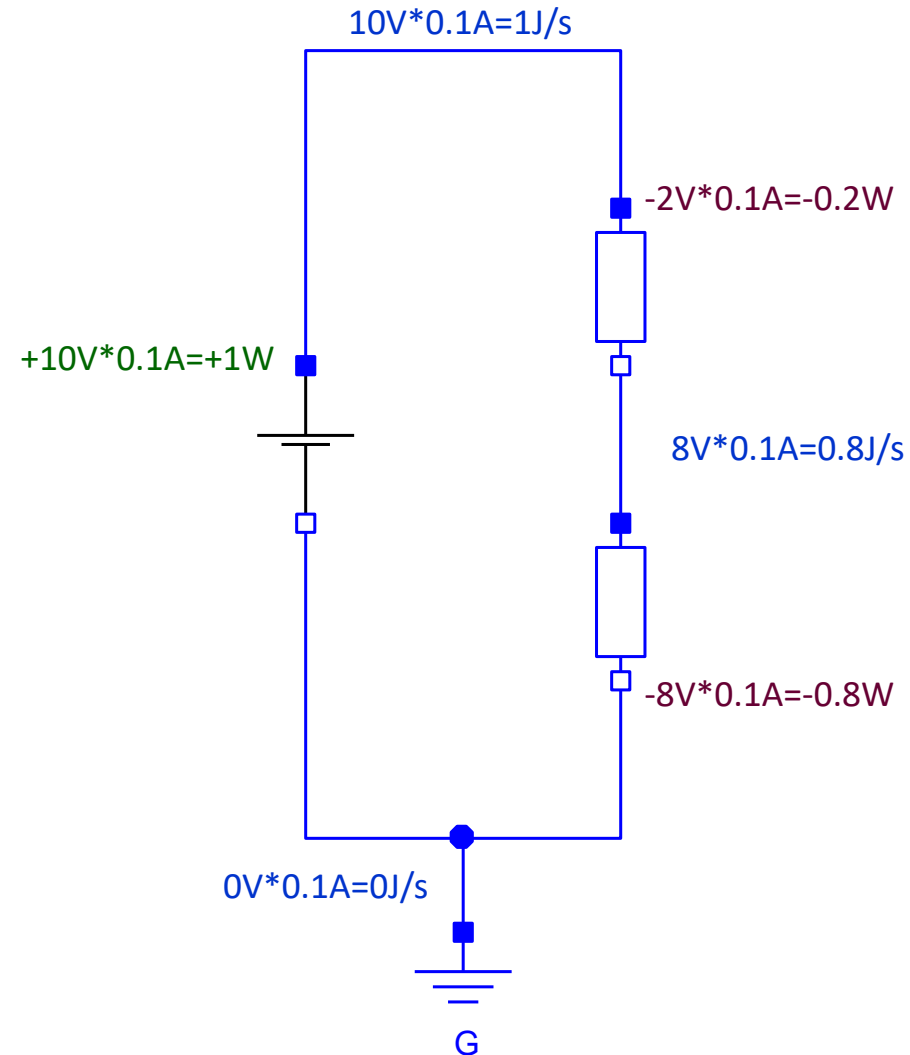
- There is a flow into the component at a certain level of energy.
- There is a flow out of the component at a possibly different level of energy.
- The difference between the two levels of energy represents **work!**
- The difference between the two flows represents **power!** (work per time)
- Energy is a potential size, whereas work represents the difference. This is the same distinction as between voltage and voltage potential.

$$v_1 = 10V, i_1 = 0.1A$$



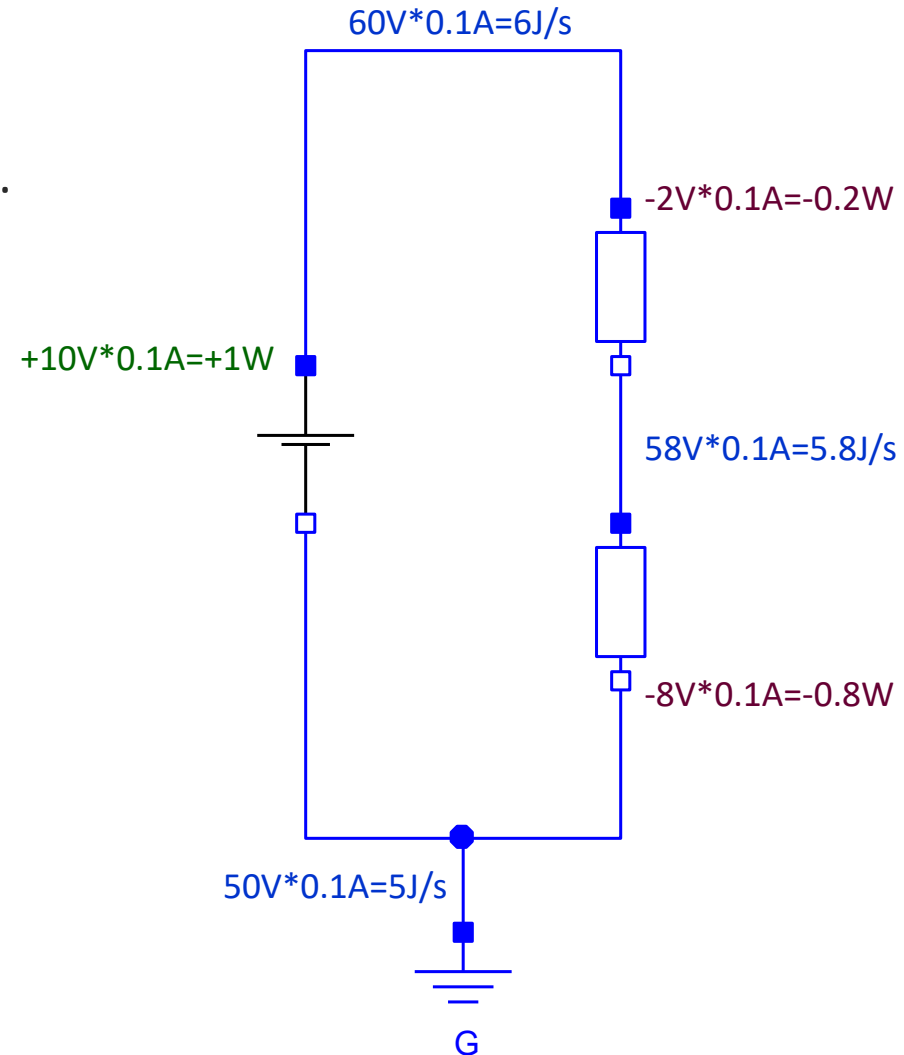
## Example for clarification

- If I change the grounding voltage of an electric circuit...



## Example for clarification

- If I change the grounding voltage of an electric circuit (from 0V to 50V)...

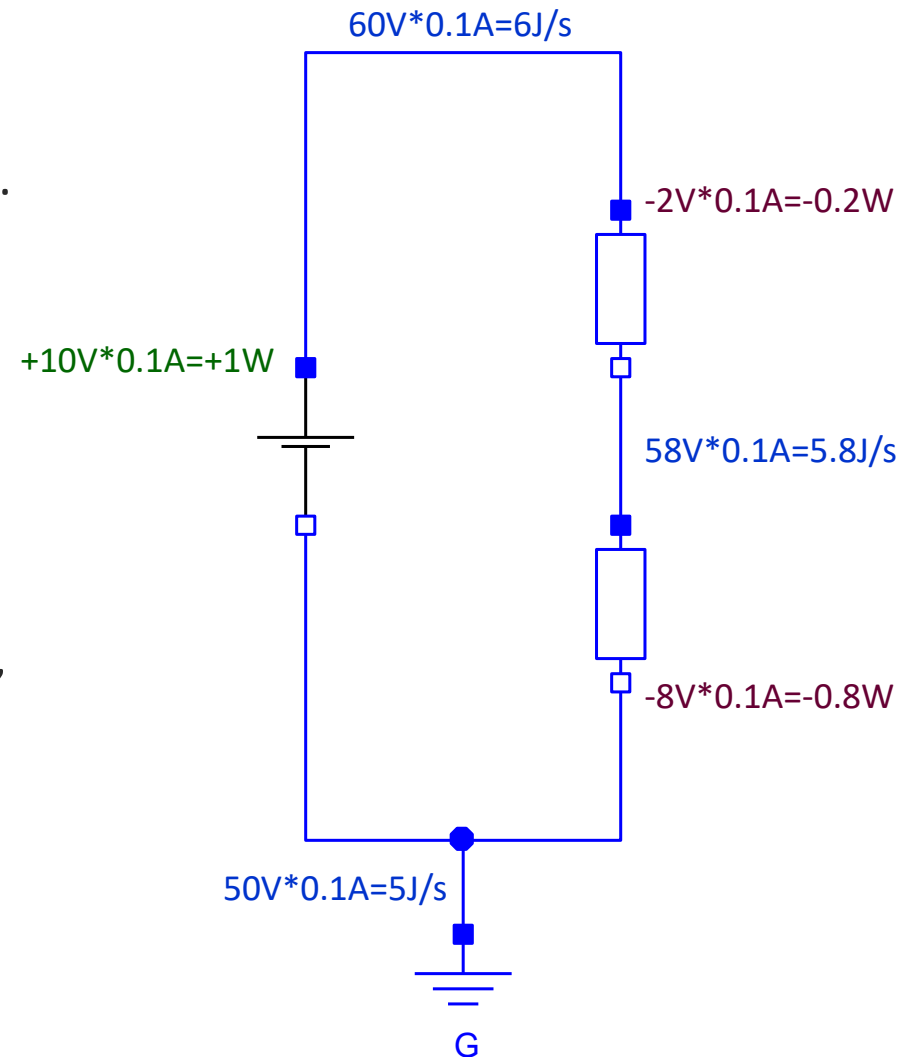




## Example for clarification

- If I change the grounding voltage of an electric circuit (from 0V to 50V)...
- ...all energy flows at the connector change.
- But the power across the components remains the same!
- Potential variables are auxiliary variables. For the physical behavior, only the difference between potentials does matter.

*(There are exceptions where the potentials cannot be chosen arbitrarily)*



Some components dissipate energy



Resistor

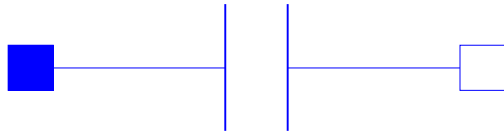
$$u = R \cdot i$$



Damper

$$\Delta v = D^{-1} \cdot f$$

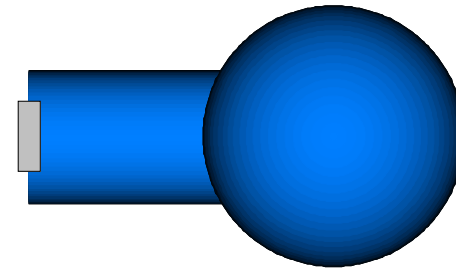
Some components store energy (by integrating the flow variable):



Capacitor

$$du/dt \cdot C = I$$

(Storage of charge)



Mass

$$dv/dt \cdot M = f \text{ (Newton's Law)}$$

(Storage of kinetic energy)

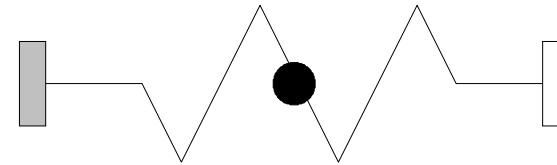
Some components store energy (by integrating the potential variable):



Inductance

$$di/dt \cdot L = u$$

*(The energy is stored in the magnetic field)*



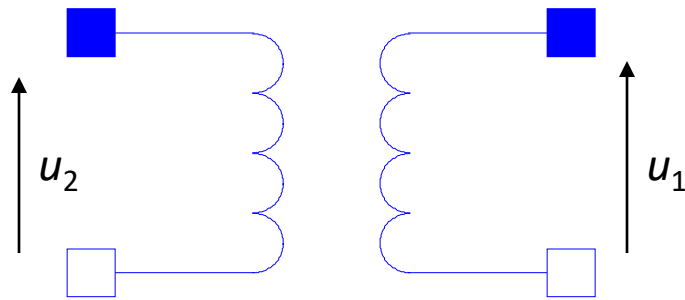
Spring

$$df/dt \cdot C^{-1} = \Delta v$$

*(Velocity is integrated to position)*

(this is not a good analogy, though)

Some components transform energy

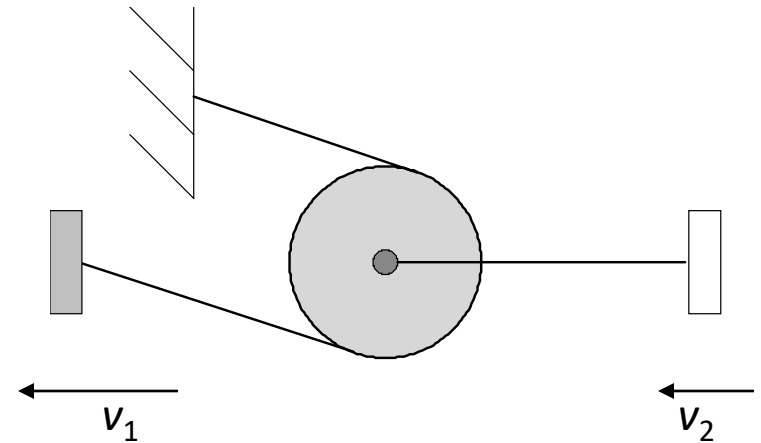


M

Transformer

$$u_2 = M \cdot u_1$$

$$M \cdot i_2 = i_1$$

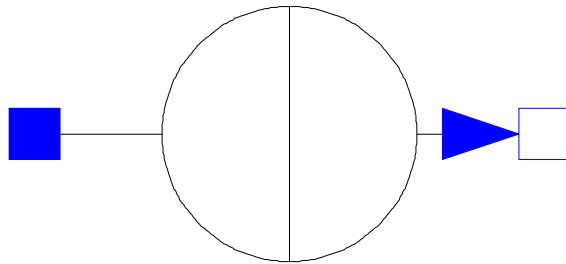


Linkage

$$v_2 = G \cdot v_1$$

$$G \cdot f_2 = f_1$$

Some components represent a source or sink of energy



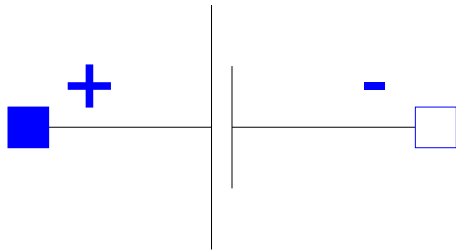
Current Source

$$i = I_0$$

Constant force

$$f = f_0$$

Some components represent a source or sink of energy



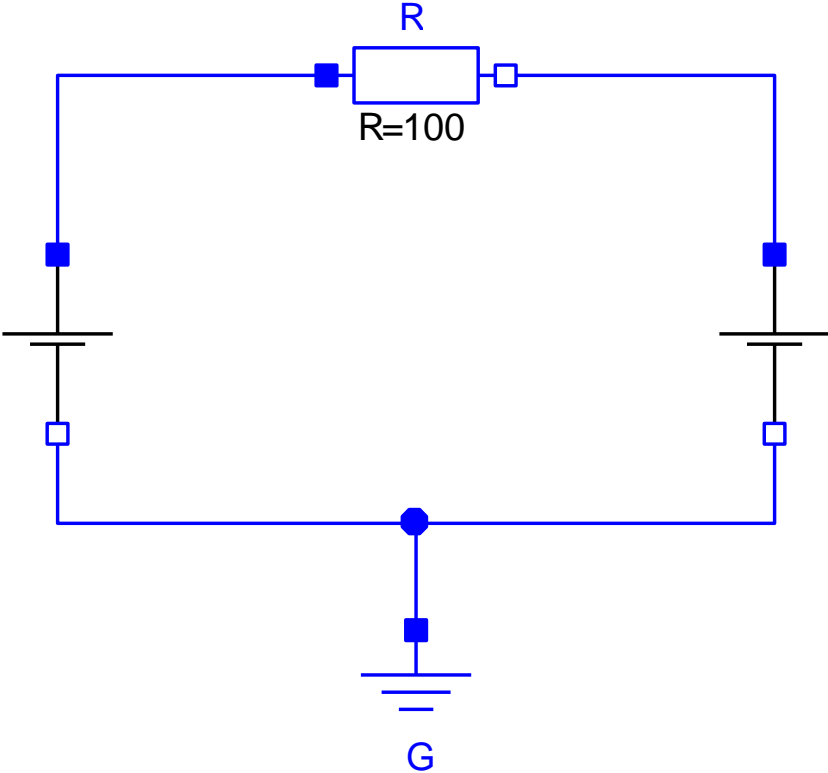
Voltage Source

$$u = U_0$$

Constant velocity

$$\Delta v = V_0$$

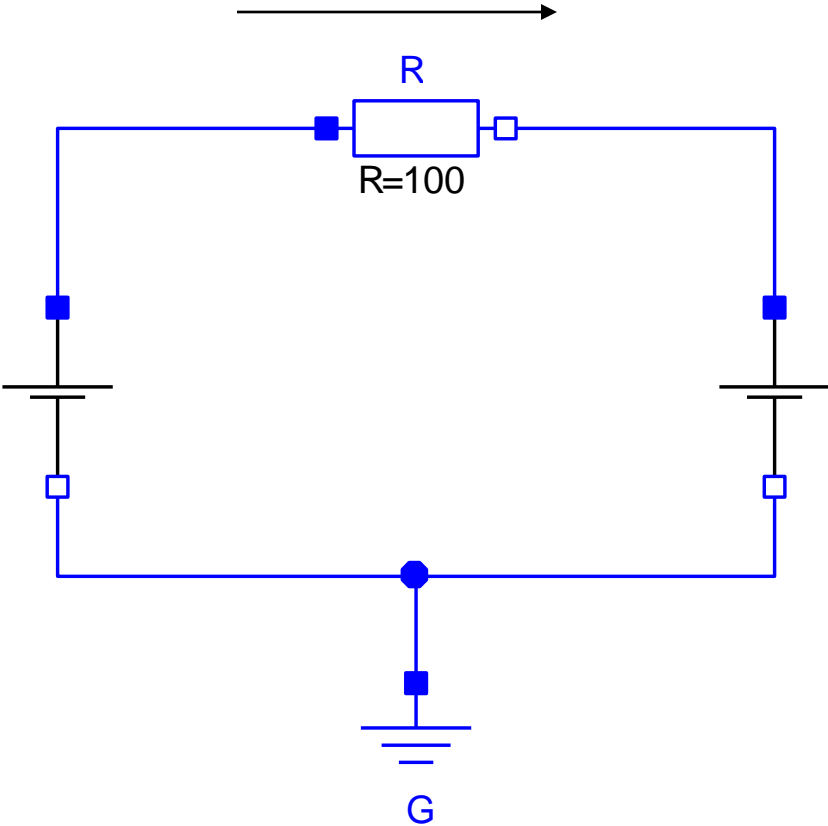
# Sink or Source?





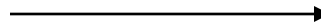
# Sink or Source?

The current flows in this direction



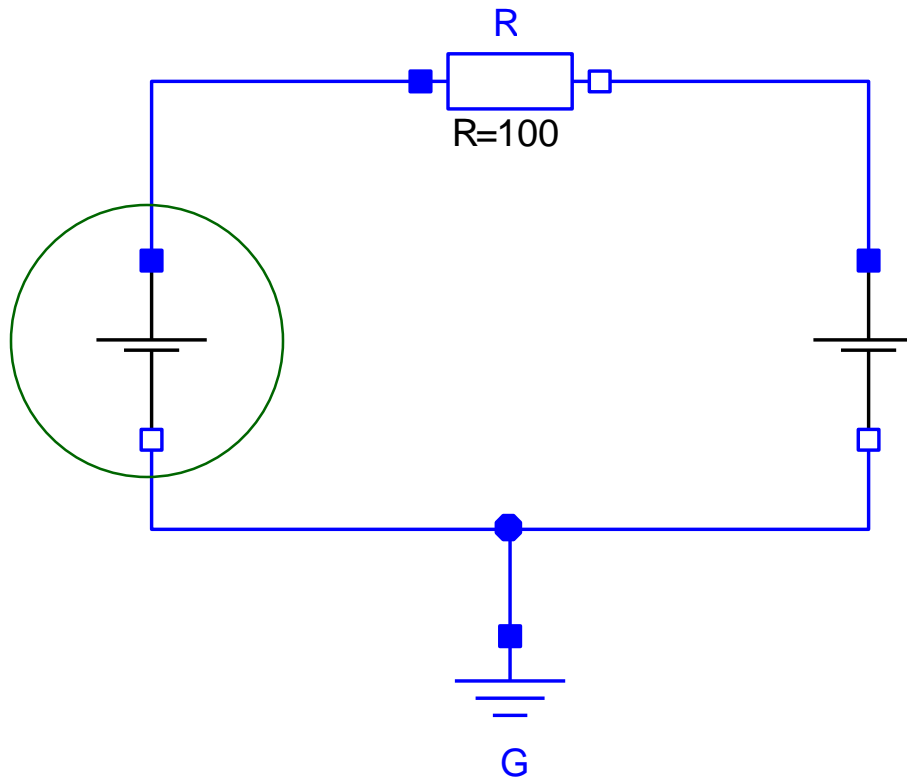
# Sink or Source?

The current flows in this direction



Here, the current flows against the gradient of the potential voltage

→ Source



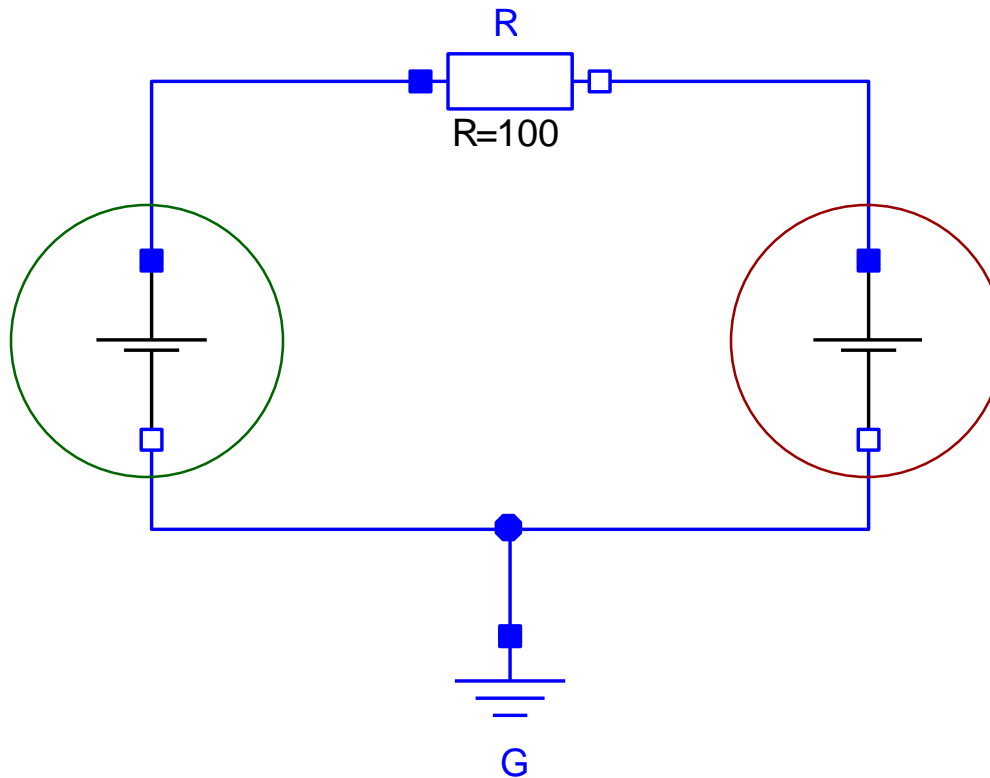
# Sink or Source?

The current flows in this direction



Here, the current flows against the gradient of the potential voltage

→ Energy Source



Here, the current flows along the gradient of the potential voltage

→ Energy Sink

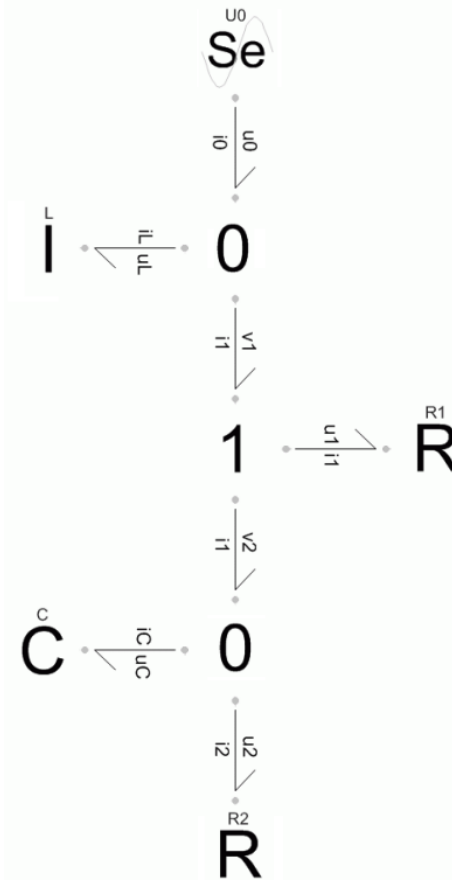
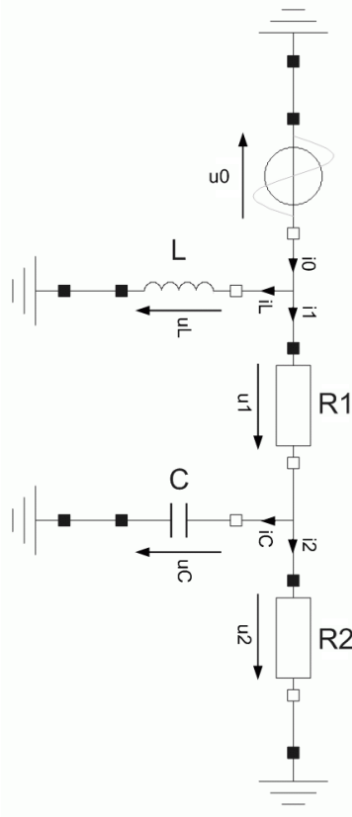
→ A source of voltage is not necessarily a source of energy!

We have seen that mechanical and electrical systems can be modeled the same way

- What about other physical domains?
- Can Kirchhoff's Laws be generalized for the complete field of thermodynamics?

The answer is **bond graphs**.

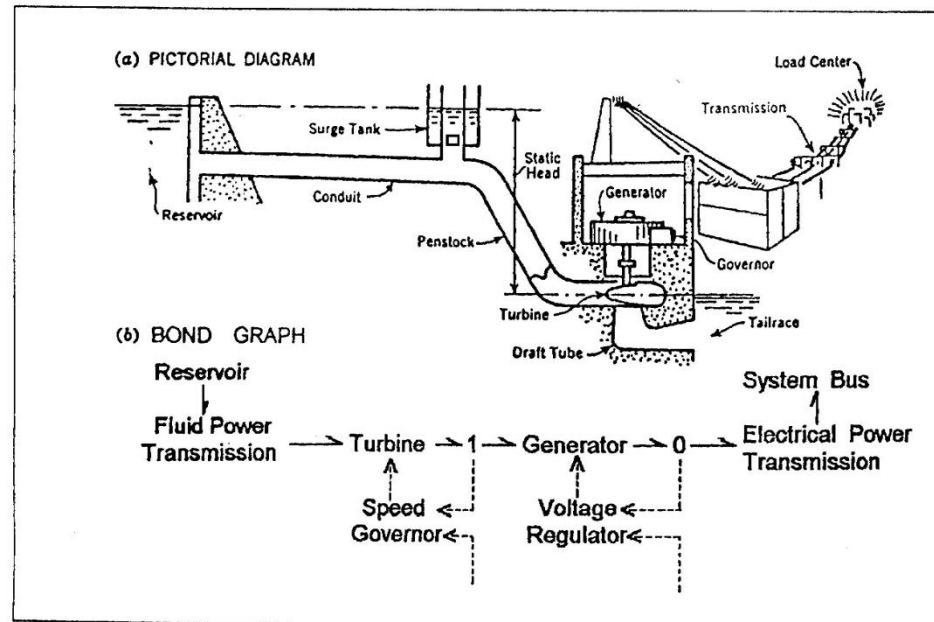
- Here the complete system is abstracted by energy-flows.



For each physical domain, there is a specific pair of effort / flow variables

Domain	Potential	Flow
Translational Mechanics	Velocity: $v$ [m/s]	Force: $f$ [N]
Rotational Mechanics	Angular Velocity: $\omega$ [1/s]	Torque: $\tau$ [Nm]
Electrics	Voltage Potential $v$ [V]	Current $i$ [A]
Magnetics	Magnetomotive Force: $\Theta$ [A]	Time-derivative of Magnetic Flux: $\dot{\Phi}$ [V]
Hydraulics	Pressure $p$ [Pa]	Volume flow rate $\dot{V}$ [m <sup>3</sup> /s]
Thermal	Temperature $T$ [K]	Entropy Flow Rate $\dot{S}$ [J/Ks]
Chemical	Chemical Potential: $\mu$ [J/mol]	Molar Flow Rate $\nu$ [mol/s]

Bond graphs have been invented by Henry M. Paynter on April 24, 1959



*Hydroelectric plant.*

- Again, an actually trivial generalization of Kirchhoff's laws took more than a century to be developed.

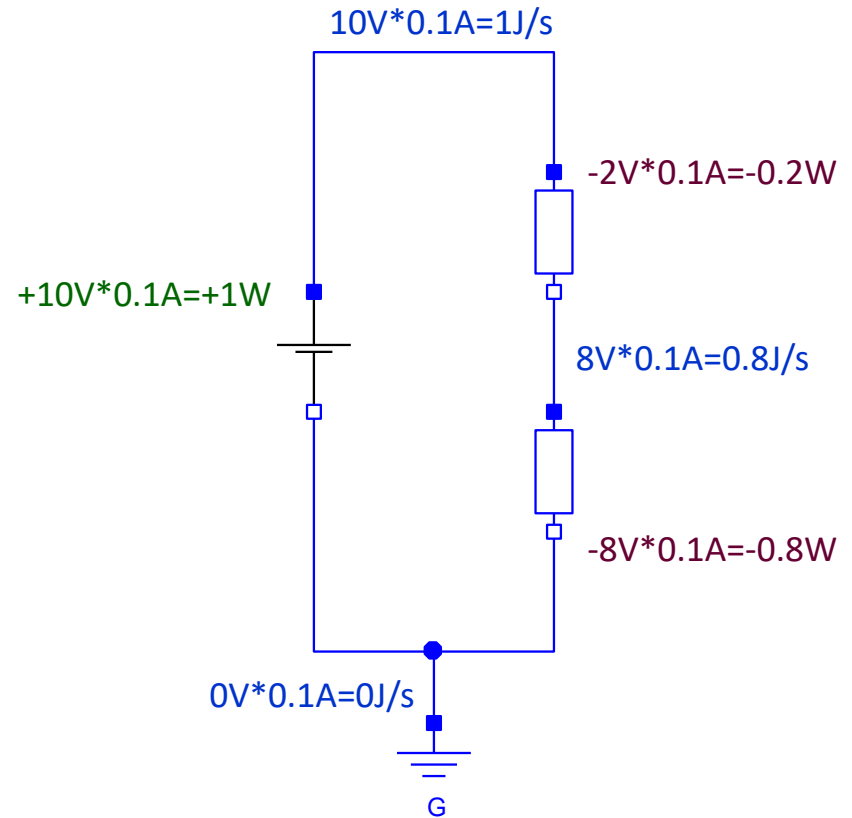
In this lecture, bond graphs are not the matter of subject, but we can profit from the major principle that underpins this methodology.

- For all physical domains, there is a correspondent pair of connector variables. Their product represents a flow of energy.
- The components all exhibit a certain energetic behavior.
- In this way, we do not have to acquire the physical knowledge domain by domain. Instead we apply the general principles of thermodynamics.



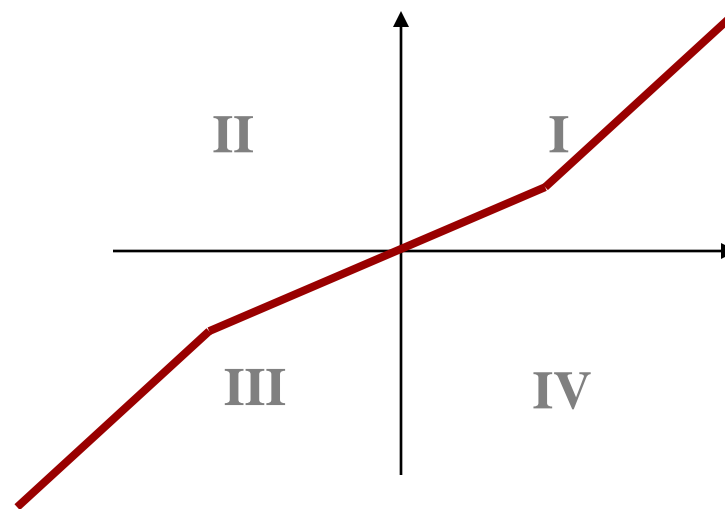
By modeling with energy flows, we can profit from the general laws of thermodynamics.

- The first law of thermodynamics states that within a closed system, the total amount of energy remains constant.
- This means that the sum of all powers quantities across the components must be zero.



Any dissipative component represents a relation that relates the flow  $F$  with a difference of potentials  $\Delta P$ .

$$\Delta P = f(F)$$



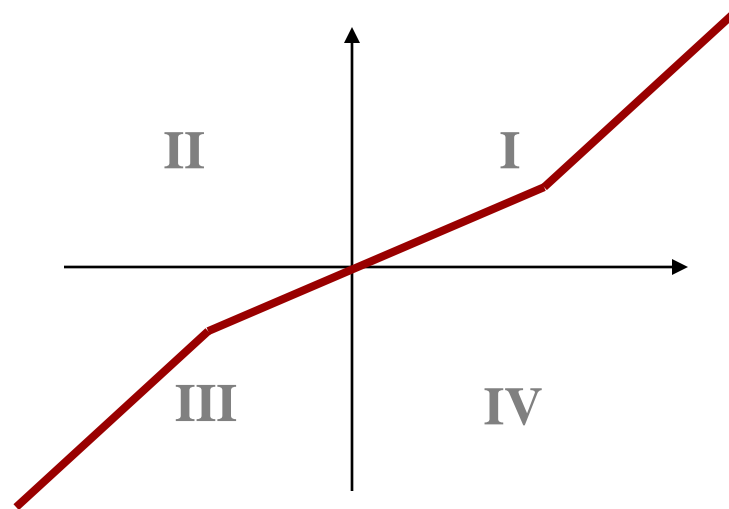
- The corresponding function  $f(\dots)$  must be located in first and third quadrant (and cross the origin).

Any storage component relates one of the two variables with the time-derivative of its partner.

$$d\Delta P/dt = f(F)$$

or

$$dF/dt = f(\Delta P)$$



- Also here: the corresponding function  $f(\dots)$  is typically located in the first and third quadrant (but not mandatory)

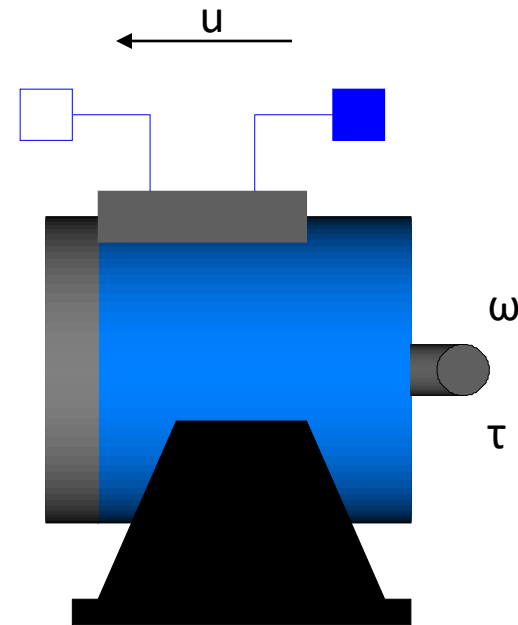
Using energy flows, we can also model across multiple domains

- An electrical engine represents a transformer from electrical energy to mechanic (rotational) energy. Energy is conserved.

$$\tau = K \cdot i$$

$$K \cdot \omega = u$$

- K is the Motor-Torque Constant

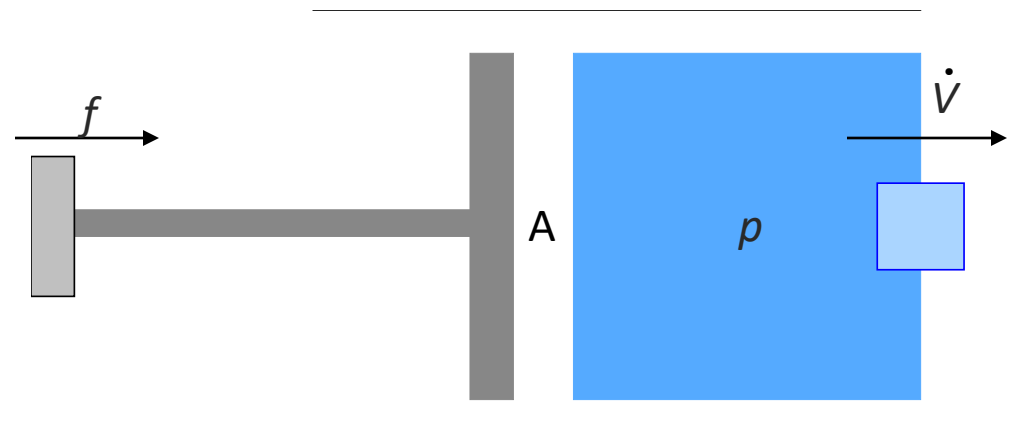


Using energy flows, we can also model across multiple domains

- A piston represents a transformer (more precisely: a gyrator) from the mechanical domain into the hydraulic domain. Also here, energy is conserved.

$$p \cdot A = f$$

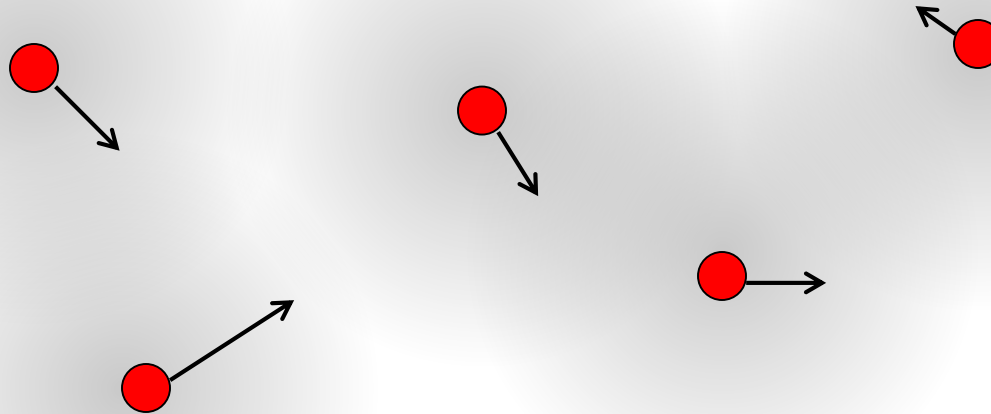
$$\dot{V} = v \cdot A$$



- $A$  is the area of the piston

# A simplistic(!) view on physics

- Simplistically, we could regard our world as being composed out of particles
- These particles create symmetric potential fields belonging to various domains.
- The gradients in the sum of these fields determine the particles acceleration.



- Then all domain interaction has to be established by particle motion (moving from one potential field of domain X into the potential field of domain Y, thereby transferring its kinetic energy between the domains)
- The kinetic energy is needed to formulate the conservation of energy:

$$E_{kin} + \sum_{domains} E_{pot} = const$$

- Particles then move so that they (locally) minimize the action functional (simplified version of Hamilton's principle)

$$\int E_{kin} - \sum_{domains} E_{pot} dt \stackrel{!}{=} min$$

- Such particle systems have an easy structure that can be formulated directly in state-space form.

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & x & x & \\ & & & x & x & \\ & & & x & x & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

- Yet it is often (typically always) infeasible to simulate such systems:
  - Too many particles,
  - Too high frequencies
  - Too dense incidence matrix



- In order to derive useful equations, one applies one or more of the following methods of idealization:
  - Scratch a domain on no interest (and the system may become dissipative)
  - Limit the interactions between particles
    - ➔ Everything with everything is replaced by a graph
  - Aggregate several particles into one representative element
    - ➔ Follow one element that represents many (Lagrangian framework)
    - ➔ Flow of many through one element (Eulerian framework)
    - In any case, the aggregate element may now have aggregate properties like (temperature, rotational orientation, etc.)
  - Fast dynamics are replaced by a steady state assumptions (such as vibrations in a steel rod or the inertia of an electron flow)

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The red lines describe the sources of (potentially problem-causing) non-linearities in the resulting equations.

- Although such a particle world is too simply to represent our world, many important concepts can be formulated within it:
  - Force,
  - Power,
  - Work and Energy,
  - Entropy.
- These concepts prove to be useful beyond our simplistic particle world and are transferred to quantum physics and the theory of relativity.
- Hence also our modeling approach can include such phenomena.

- Ideally, any form of energy can be completely transformed into any other. (Practically, all transformations involve dissipation.)
- The dissipation of energy represents the transformation of energy into thermal energy.
- But there is one important exception: The 2<sup>nd</sup> law of thermodynamics states that entropy can only increase.
- The thermal domain possesses the flow of entropy as connector variable. This means, that for any thermal sub-system the inflow must be equal or greater than the outflow.

- Thermal energy can only be transformed into other forms of energy up to a limited extent.
- In order to transform thermal energy into any other form, we need a temperature gradient between two reservoirs  $T_{\text{cold}}$  and  $T_{\text{hot}}$ .
- The precise limit of the efficiency is determined by the Carnot Factor. This is the 3<sup>rd</sup> law of Thermodynamics.

$$\eta_C = 1 - T_{\text{cold}}/T_{\text{hot}}$$

(Temperature in Kelvin)

- Since  $T_{\text{cold}} > 0 \rightarrow \eta_C < 1$

- All physical connections can be represented by a pair of a potential variable and a flow variable whose product represents energy flow.
- Using this knowledge, the equations for the connections can be automatically generated.
- All components exhibit a certain energetic behavior. Once we understand the energetic behavior, we can apply it in various physical domains.
- Interaction between domains is represented by a transformation of energy.

- Next week, we are going to learn how to punch all this into a computer!
- Don't worry if you haven't understood every single component equation. We will look at the modeling of electrical and mechanical systems in depth.

**Questions ?**